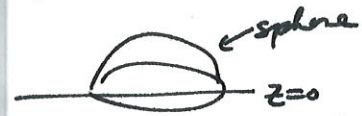
MAT 201, Quiz #8, Fall 2016 Name KEY

Instructions: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Use Stokes' Theorem to evaluate $\int\int_S \vec{\nabla} \times \vec{F} \cdot d\vec{S}$ for $\vec{F}(x,y,z) = 2y\cos z\,\hat{\imath} + e^x \sin z\,\hat{\jmath} + xe^y\,\hat{k}$ where S is the hemisphere $x^2 + y^2 + z^2 = 4, z \geq 0$, oriented upward.

$$\vec{\nabla}\times\vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 2y\cos z & e^x\sin z & xe^y \end{vmatrix} =$$



intersects at $x^2+y^2=4$

$$(xe^y - e^x\cos z)\hat{\imath} - (e^y + 2y\sin z)\hat{\jmath} + (e^x\sin z - 2\cos z)\hat{k}$$

$\hat{dS} = \hat{k}$, $z=0$

$$(\vec{\nabla}\times\vec{F})\cdot d\vec{S} = e^x\sin z - 2\cos z \quad \text{at } z=0 \Rightarrow e^x(0) - 2(1) = -2$$

$$\iint -2\, dA = \int_0^{2\pi}\int_0^2 -2r\,dr\,d\theta = \int_0^{2\pi} -r^2 \Big|_0^2 d\theta = -4\int_0^{2\pi} d\theta =$$

$$\boxed{-8\pi}$$

2. Find the unit tangent vector and the unit normal vector for $\vec{r}(t) = 3t\hat{\imath} + 4\sin t\,\hat{\jmath} + 4\cos t\,\hat{k}$.

$$r'(t) = 3\hat{\imath} + 4\cos t\,\hat{\jmath} - 4\sin t\,\hat{k} \qquad \|r'(t)\| = \sqrt{3^2 + 16\cos^2 t + 16\sin^2 t} = \sqrt{9+16} = \sqrt{25} = \sqrt{5}$$

$$\boxed{T(t) = \tfrac{3}{5}\hat{\imath} + \tfrac{4}{5}\cos t\,\hat{\jmath} - \tfrac{4}{5}\sin t\,\hat{k}}$$

$$T'(t) = 0\hat{\imath} - \tfrac{4}{5}\sin t\,\hat{\jmath} - \tfrac{4}{5}\cos t\,\hat{k} \qquad \|T'(t)\| = 4/5$$

$$\boxed{N(t) = -\sin t\,\hat{\jmath} - \cos t\,\hat{k}}$$

3. Find an equation of the tangent plane to the surface $\vec{r}(u,v) = u^2\hat{\imath} + 2u\sin v\,\hat{\jmath} + u\cos v\,\hat{k}$ where $u = 1, v = 0$.

$(1, 0, 1)$

$$\vec{r}_u = 2u\hat{\imath} + 2\sin v\,\hat{\jmath} + \cos v\,\hat{k} \Rightarrow 2\hat{\imath} + 0\hat{\jmath} + 1\hat{k}$$

$$\vec{r}_v = 0\hat{\imath} + 2u\cos v\,\hat{\jmath} + u\sin v\,\hat{k} \Rightarrow 0\hat{\imath} + 2\hat{\jmath} + 0\hat{k}$$

$$\begin{vmatrix} \hat{\imath} & \hat{\jmath} & k \\ 2 & 0 & 1 \\ 0 & 2 & 0 \end{vmatrix} = \vec{r}_u \times \vec{r}_v$$

$$= (0-2)\hat{\imath} - (2-0)\hat{\jmath} + (4-0)\hat{k}$$

$\langle -2, 0, 4\rangle$

$$-2(x-1) \qquad +4(z-1) = 0$$