MAT 201, Quiz #7, Fall 2016   Name _____ KEY _____

Instructions: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Set up a triple integral for the solid enclosed by $x^2 + z^2 = 4, y = -1, y + z = 4$.

$$\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{-1}^{4-z} dy\, dz\, dx \qquad \textit{answers may vary}$$

2. Evaluate $\int_{-2}^{2} \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} \int_{\sqrt{x^2+y^2}}^{2} xz\, dz\, dx\, dy$ by switching to cylindrical or spherical coordinates.

$$\int_{0}^{2\pi} \int_{0}^{2} \int_{r}^{2} zr^2 \cos\theta\, dz\, dr\, d\theta \qquad \textit{cylindrical}$$

3. Use the Fundamental Theorem of Line Integrals to evaluate $\int_C 2xe^{-y}dx + (2y - x^2e^{-y})dy$ on any path from $(1,0)$ to $(2,1)$. Be sure to confirm that the field is conservative.

$$\frac{\partial M}{\partial y} = -2xe^{-y}$$

$$\frac{\partial N}{\partial x} = -2xe^{-y}$$

Conservative

$$\int 2xe^{-y}dx = x^2e^{-y} + g(y)$$

$$\int 2y - x^2e^{-y}dy = y^2 + x^2e^{-y} + h(x)$$

$$f(x,y) = x^2e^{-y} + y^2 + K$$

$$f(2,1) - f(1,0) = 2^2e^{-1} + 1^2 - (1^2e^0 + 0^2) =$$

$$\frac{4}{e} + 1 - 1 + 0 = \boxed{\frac{4}{e}}$$