Name _____ KEY

**Instructions**: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.
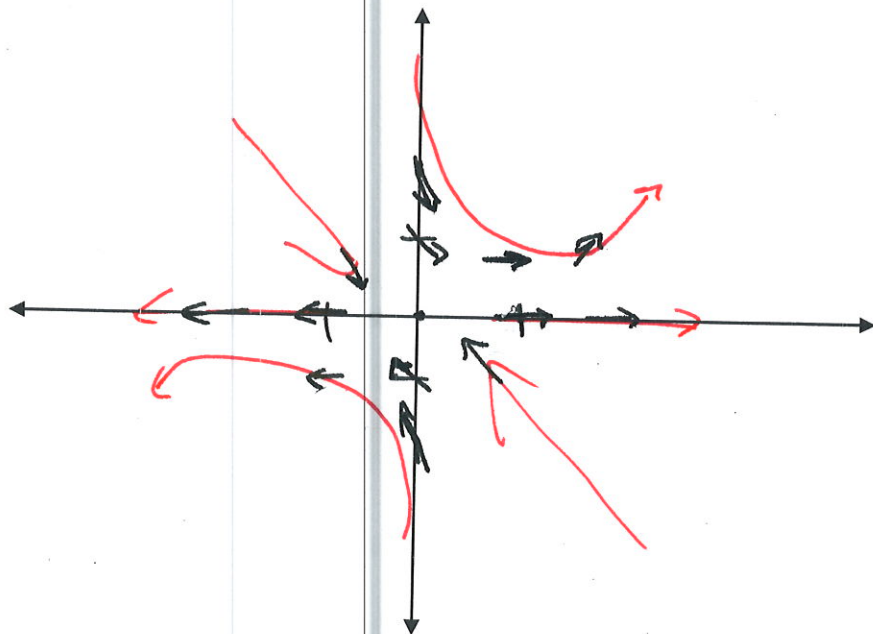
1. Find $\lim\limits_{(x,y)\to(0,0)} \dfrac{5y^4\cos^2 x}{x^4+y^4}$.

let $y = kx$

$$\lim_{x\to 0} \frac{5k^4 x^4 \cos^2 x}{x^4 + k^4 x^4} = \lim_{x\to 0} \frac{x^4(5k^4\cos^2 x)}{x^4(1+k^4)} = \frac{5k^4(1)}{1+k^4}$$

Since limit depends on $k$, DNE

2. Sketch the vector field
$\vec{F}(x,y) = y\hat{i} + (x-y)\hat{j}$.
Plot enough points to see the general behavior of the field.

| $(x,y)$ | $y\hat{i}$ | $\hat{j}(x-y)$ |
|---------|------|------|
| $(0,0)$ | $0$ | $0$ |
| $(1,0)$ | $0$ | $1$ |
| $(0,1)$ | $1$ | $-1$ |
| $(-1,0)$ | $0$ | $-1$ |
| $(0,-1)$ | $-1$ | $1$ |
| $(1,1)$ | $1$ | $0$ |
| $(-1,-1)$ | $-1$ | $0$ |
| $(-1,1)$ | $1$ | $-2$ |
| $(1,-1)$ | $-1$ | $2$ |
| $(2,1)$ | $1$ | $1$ |



3. Find an equation in rectangular coordinates for $\vec{r}(u,v) = (1-u^2-v^2)\hat{i} - v\hat{j} - u\hat{k}$, and describe the surface.

$y = -v$, $z = -u$ $\Rightarrow$ $v = -y$, $u = -z$

$x = 1 - (-z)^2 - (-y)^2 \Rightarrow$

$x = 1 - z^2 - y^2$    Paraboloid around x-axis