MAT 201, Quiz #13, Fall 2016 Name KEY

Instructions: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Find the center of mass with $\rho(x,y) = \sqrt{x^2+y^2}$ in the region bounded by $y = 4 - x$ between $x = 0, y = 0$.

$\theta = [0, \pi/2]$ $\rho = r$

$r\sin\theta = 4 - r\cos\theta$

$r\sin\theta + r\cos\theta = 4$

$r = \frac{4}{\sin\theta + \cos\theta}$

$$M = \int_0^{\pi/2}\int_0^{\frac{4}{\sin\theta+\cos\theta}} r\cdot r\cdot dr\,d\theta$$

$$= \int_0^{\pi/2}\int_0^{\frac{4}{\sin\theta+\cos\theta}} r^2\,dr\,d\theta = \frac{1}{3}\int_0^{\pi/2}\left(\frac{4}{\sin\theta+\cos\theta}\right)^3 d\theta \approx 17.3144$$

$x = r\cos\theta$

$$M_y = \int_0^{\pi/2}\int_0^{\frac{4}{\sin\theta+\cos\theta}} r^3\cos\theta\,dr\,d\theta = \frac{1}{4}\int_0^{\pi/2}\left(\frac{4}{\sin\theta+\cos\theta}\right)^4\cos\theta\,d\theta \approx 25.9716$$

$y = r\sin\theta$

$$M_x = \int_0^{\pi/2}\int_0^{\frac{4}{\sin\theta+\cos\theta}} r^3\sin\theta\,dr\,d\theta = \frac{1}{4}\int_0^{\pi/2}\left(\frac{4}{\sin\theta+\cos\theta}\right)^4\sin\theta\,d\theta \approx 25.9716$$

$$\bar{x} = \frac{M_y}{M} = \frac{25.9716}{17.3144} \approx 1.5 \qquad \bar{y} = \frac{25.9716}{17.3144} \approx 1.5 \qquad (3/2, 3/2)$$

2. A probability density function is given by $f(x,y) = cx^2y^3$ on $[0,2]\times[0,3]$.

a. Find c.

$$\int_0^2\int_0^3 cx^2y^3\,dy\,dx = \int_0^2 \frac{c}{4}x^2y^4\,dx\Big|_0^3 = \int_0^2 \frac{81c}{4}x^2\,dy = \frac{27c}{4}x^3\Big|_0^2 = 54c$$

$$c = 1/54$$

b. Find $P(X \le 1, Y \le 1)$

$$\int_0^1\int_0^1 \frac{1}{54}x^2y^3\,dy\,dx = \int_0^1 \frac{1}{54}\cdot\frac{1}{4}x^2y^4\Big|_0^1 = \frac{1}{216}\int_0^1 x^2\,dx = \frac{1}{648}x^3\Big|_0^1 = \frac{1}{648}$$

c. Find $P(X \le Y)$

$$\int_0^2\int_0^x \frac{1}{54}x^2y^3\,dy\,dx = \int_0^2 \frac{1}{54}\cdot\frac{1}{4}x^2y^4\Big|_0^x dx = \frac{1}{216}\int_0^2 x^6\,dx = \frac{1}{1512}\cdot x^7\Big|_0^2$$

$$= \frac{128}{1512} = \frac{16}{189}$$