MAT 201, Quiz #11, Fall 2016 Name KEY

Instructions: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Describe the difference between an orientable surface and a non-orientable surface.

An orientable surface has a normal vector oriented uniquely for every point in space.

A non-orientable surface has two or more different normal vectors at the same point.

2. Find the surface integral for $\int\int_S xzdS$ where S is the part of the plane $2x+2y+z=4$ in the [redacted]t octant.
first

$dS = \sqrt{4+4+1}\,dA = \sqrt{9}\,dA = 3\,dA$

$z=0 \Rightarrow \frac{2x+2y=4}{2}$

$x+y=2$

$y=2-x$

$z=4-2x-2y$

$$\int_0^2\int_0^{2-x} x(4-2x-2y)\cdot 3\,dy\,dx = 3\int_0^2 4xy-2x^2y-xy^2\Big|_0^{2-x}dx$$

$4x-2x^2-2xy$

$$3\int_0^2 4x(2-x)-2x^2(2-x)-x(2-x)^2\,dx$$

$$3\int_0^2 8x-4x^2-4x^2+2x^3-x(4-4x+x^2)\,dx = 3\int_0^2 8x-8x^2+2x^3-4x+4x^2-x^3\,dx$$

$$=3\int_0^2 4x-4x^2+x^3\,dx = 3\left[2x^2-\tfrac{4}{3}x^3+\tfrac{1}{4}x^4\Big|_0^2\right] = 3\left[8-\tfrac{32}{3}+4\right] = \boxed{4}$$

3. Use the Divergence Theorem to find the flux through the region of the sphere with center at the origin of radius 2 through the field $\vec{F}(x,y,z) = (x^3+y^3)\hat{\imath} + (y^3+z^3)\hat{\jmath} + (z^3+z^3)\hat{k}$.

$\vec{\nabla}\cdot\vec{F} = 3x^2+3y^2+3z^2$ $\quad S: x^2+y^2+z^2=4$

$$\iiint \operatorname{div} F\,dV = \int_0^{\pi}\int_0^{2\pi}\int_0^{2} 3\rho^2\cdot\rho^2\sin\varphi\,d\rho\,d\theta\,d\varphi$$

$$\int_0^{\pi}\int_0^{2\pi}\int_0^2 3\rho^4\sin\varphi\,d\rho\,d\theta\,d\varphi = \int_0^{\pi}\int_0^{2\pi}\tfrac{3}{5}\rho^5\Big|_0^2\sin\varphi\,d\theta\,d\varphi$$

$$2\pi\cdot\frac{96}{5}\int_0^{\pi}\sin\varphi\,d\varphi = \frac{192}{5}\pi\left(-\cos\varphi\Big|_0^{\pi}\right) = \frac{192\pi}{5}(1+1) = \boxed{\frac{384\pi}{5}}$$