MAT 201, Quiz #10, Fall 2016 Name KEY

Instructions: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Find the arc length of $\vec{r}(t) = t\hat{\imath} + \frac{1}{2}t^2\hat{\jmath} + t^2\hat{k}$ on $[1,4]$. Set up the integral; you may integrate numerically.

$r'(t) = 1\hat{i} + t\hat{j} + 2t\hat{k}$ $\quad \|r'(t)\| = \sqrt{1+t^2+4t^2} = \sqrt{1+5t^2}$

$\int_1^4 \sqrt{1+5t^2}\, dt \approx 17.0755$

2. Calculate the curvature of $\vec{r}(t) = t\hat{\imath} + \frac{1}{2}t^2\hat{\jmath} + \frac{1}{3}t^3\hat{k}$ at the point $t = 2$. What is the radius of curvature?

$r'(t) = 1\hat{i} + t\hat{j} + t^2\hat{k}$ $\quad \|r'(t)\| = \sqrt{1+t^2+t^4}$

$r''(t) = 0\hat{i} + 1\hat{j} + 2t\hat{k}$

$K = \dfrac{\|r'(t) \times r''(t)\|}{\|r'(t)\|^3} = \dfrac{\sqrt{t^4+4t^2+1}}{(1+t^2+t^4)^{3/2}}$

$r'(t) \times r''(t) =$

$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & t & t^2 \\ 0 & 1 & 2t \end{vmatrix} = (2t^2-t^2)\hat{i} - (2t-0)\hat{j} + (1-0)\hat{k}$

$t^2\hat{i} - 2t\hat{j} + 1\hat{k}$

$t=2 \quad K(2) = \dfrac{\sqrt{16+16+1}}{(1+4+16)^{3/2}} =$

$\|r'(t) \times r''(t)\| = \sqrt{t^4+4t^2+1}$

$\boxed{K = \dfrac{\sqrt{33}}{(\sqrt{21})^3}} \quad R = \dfrac{(\sqrt{21})^3}{\sqrt{33}}$

3. Find the surface area of the part of the paraboloid $z = 4 - x^2 - y^2$ that lies above the xy-plane.

$F = 4-x^2-y^2-z \quad \nabla F = \langle -2x, -2y, -1 \rangle$

$\iint_R \sqrt{4x^2+4y^2+1}\, dA = \int_0^{2\pi}\int_0^2 r\sqrt{4r^2+1}\, dr\, d\theta =$

$u = 4r^2+1$

$du = 8r\, dr$

$\frac{1}{8}du = r\, dr$

$\int u^{1/2} du$

$r=0 \Rightarrow u=1$

$r=2 \Rightarrow u=17$

$\int_0^{2\pi} \frac{1}{8} \cdot \frac{2}{3} u^{3/2} \Big|_1^{17} = \int_0^{2\pi} \frac{1}{12}(17^{3/2}-1)\, d\theta =$

$\frac{\pi}{6}(17^{3/2}-1)$