201 Homework #3 Key

1. a. $x^2+y^2+z^2=10$

cylindrical $r^2+z^2=10$

spherical $\rho^2=10 \Rightarrow \rho=\sqrt{10}$

Sphere radius $\sqrt{10}$

b. $x^2+y^2=9$

cylindrical $r^2=9 \Rightarrow r=3$

spherical $\rho^2\sin^2\varphi=9$

$\Rightarrow \rho\sin\varphi=3$

$\Rightarrow \boxed{\rho=3\csc\varphi}$

Cylinder radius 3

c. $x^2+y^2-3z^2=0$

$x^2+y^2=3z^2$

cylindrical $r^2=3r^2$

$\Rightarrow r=\sqrt{3}z$

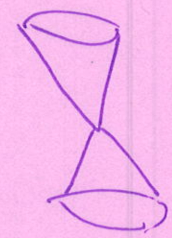
Spherical $\cancel{\rho^2}\sin^2\varphi=\sqrt{3}^2\cancel{\rho^2}\cos^2\varphi$

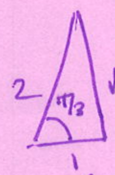
$\Rightarrow \cancel{\rho}\sin\varphi=\sqrt{3}\cancel{\rho}\cos\varphi$

$\Rightarrow \tan\varphi=\sqrt{3}$

$\varphi=\pi/3$

Cone





opens at angle from vertical of $\pi/3$

d. $y=x^2$

cylindrical $\cancel{r}\sin\theta=r^2\cos^2\theta$

$\Rightarrow \frac{\sin\theta}{\cos^2\theta}=r \Rightarrow r=\tan\theta\sec\theta$

spherical $\cancel{\rho}\sin\theta\cancel{\sin\varphi}=\rho^{\cancel{2}}\sin^{\cancel{2}}\varphi\cos^2\theta$

$\sin\theta=\rho\sin\varphi\cos^2\theta$

$\rho=\tan\theta\sec\theta\csc\varphi$

around z-axis

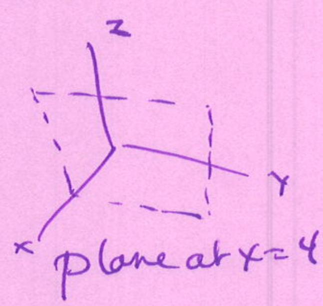
e. $x=4$

spherical $\rho\sin\varphi\cos\theta=4$

$\Rightarrow \rho=4\csc\varphi\sec\theta$

cylindrical $r\cos\theta=4$

$\Rightarrow r=4\sec\theta$



plane at $x=4$

2a. $r=2$

$\Rightarrow r^2=4$

rectangular $x^2+y^2=4$

Cylinder radius 2

201 Homework #3 key

2b. $z = r^2 \cos^2\theta$ $\qquad$ $z = x^2$ (rectangular)

around y axis

2c. $\rho = 2\sec\varphi \Rightarrow \rho\cos\varphi = 2 \Rightarrow z = 2$

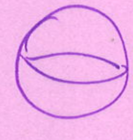
rectangular

plane at $z = 2$

d. $\rho = 2 \Rightarrow \rho^2 = 4 \Rightarrow x^2 + y^2 + z^2 = 4$

rectangular

Sphere radius 2



e. $r = 2\sin\theta$

$\Rightarrow r^2 = 2r\sin\theta \Rightarrow x^2 + y^2 = 2y$

$x^2 + (y^2 - 2y + 1) = 1$

$x^2 + (y-1)^2 = 1$

rectangular

cylinder shifted off origin by 1 w/radius 1

f. $r = \frac{1}{2}z \Rightarrow r^2 = \frac{1}{4}z^2 \Rightarrow x^2 + y^2 = \frac{1}{4}z^2$

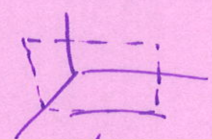
rectangular

Cone angle is about 14°

g. $\rho = 4\csc\varphi\sec\theta \Rightarrow \rho\sin\varphi\cos\theta = 4$

$x = 4$

rectangular



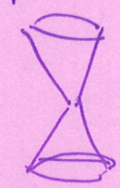
plane at $x = 4$

h. $\varphi = \pi/6$ $\qquad$ $\tan\varphi = \tan\pi/6 = 1/\sqrt{3}$

$\Rightarrow \left(\rho\sin\varphi = \frac{1}{\sqrt{3}}\rho\cos\varphi\right)^2$

$\Rightarrow \rho^2\sin^2\varphi = \frac{1}{3}\rho^2\cos^2\varphi$

$x^2 + y^2 = \frac{1}{3}z^2$



Cone

3. $z = \sqrt{2y}$ $\qquad$ $x^2 + z^2 = (\sqrt{2y})^2 \Rightarrow x^2 + z^2 = 2y$

4. $\rho = 4.366$ ly. $\quad \theta: \left(14 + \frac{39}{60} + \frac{36}{3600}\right) \times 15° \cdot \frac{\pi}{180} = 3.837979$ radians

let 0h 0min 0 sec ≡ $\theta = 0$

$\varphi: \left[90° + \left(60 + \frac{50}{60} + \frac{14}{3600}\right)\right] \cdot \frac{\pi}{180} = 2.632606$ radians

$(\rho, \varphi, \theta) = (4.366, 2.636206, 3.837979)$

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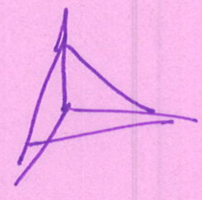
5.
a. let $u = x$, $y = v$ $\quad z = 6 - x - y \Rightarrow 6 - u - v$ $\quad$ plane



$\vec{r}(u,v) = u\hat{\imath} + v\hat{\jmath} + (6-x-y)\hat{k}$

b. $\dfrac{4x^2 + y^2 = 16}{16} \Rightarrow \dfrac{x^2}{4} + \dfrac{y^2}{16} = 1$

elliptical cylinder

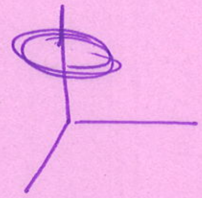
$\vec{r}(u,v) = 2\cos u\,\hat{\imath} + 4\sin u\,\hat{\jmath} + v\hat{k}$

$u \in [0, 2\pi]$

c. $\vec{r}(u,v) = v\cos u\,\hat{\imath} + v\sin u\,\hat{\jmath} + 4\hat{k}$

$v \in [0, 3]$

$u \in [0, 2\pi]$



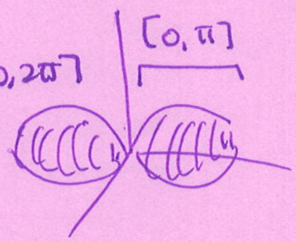
d. $x = \sin z \Rightarrow x^2 + y^2 = \sin^2 z$

let $z = v$

$\theta = u$

$\vec{r}(u,v) = \sin v\cos u\,\hat{\imath} + \sin v\sin u\,\hat{\jmath} + v\hat{k}$ $\quad u \in [0, 2\pi]$

$v \in [0, \pi)$



e. $\vec{r}(u,v) = 3\cos u\sin v\,\hat{\imath} + 2\sin u\sin v\,\hat{\jmath} + \cos v\,\hat{k}$

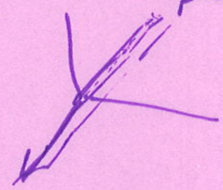
$u \in [0, 2\pi]$

$v \in [0, \pi]$

f. let $y = u$, $x = v$

$\vec{r}(u,v) = v\hat{\imath} + u\hat{\jmath} + u\hat{k}$

plane $y = z$



g. $y = x^{3/2}$

$y^2 + z^2 = [r(x)]^2 \Rightarrow y^2 + z^2 = x^3$ $\quad$ let $x = v$

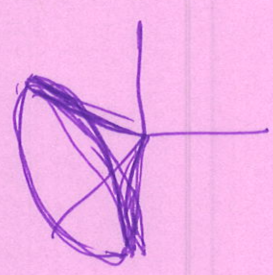
$r = v^{3/2}$

$u = \theta$ $\quad u \in [0, 2\pi)$

$v \geq 0$

$\vec{r}(u,v) =$

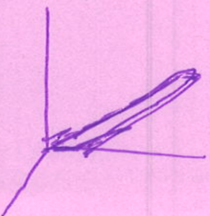
$v\,\hat{\imath} + v^{3/2}\sin u\,\hat{\jmath} + v^{3/2}\cos u\,\hat{k}$



6a. $\vec{r}(u,v) = u\hat{\imath} + v\hat{\jmath} + \frac{v}{2}\hat{k}$ $\quad z = \frac{v}{2}$ $\quad 2z = v$

$2z = y$

plane



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6b. $\vec{r}(u,v) = 2u\cos v + 2u\sin v\,\hat{j} + \frac{1}{2}u^2\hat{k}$

$x^2+y^2 = 4u^2\cos^2 v + 4u^2\sin^2 v = 4u^2$

$x^2+y^2 = 8z$ paraboloid

$z = \frac{1}{2}u^2$
$8z = 4u^2$

c. $\vec{r}(u,v) = 3\cos v\cos u\,\hat{i} + 3\cos v\sin u\,\hat{j} + 5\sin v\,\hat{k}$

ellipsoid $\frac{x^2}{9} + \frac{y^2}{9} + \frac{z^2}{5} = 1$

(roles of sine & cosine flipped from standard spherical)

d. $\vec{r}(u,v) = 4\cos u\,\hat{i} + 4\sin u\,\hat{j} + v\hat{k}$

$x^2+y^2 = 16\cos^2 u + 16\sin^2 u = 16$

$x^2+y^2=16$

cylinder of radius 4

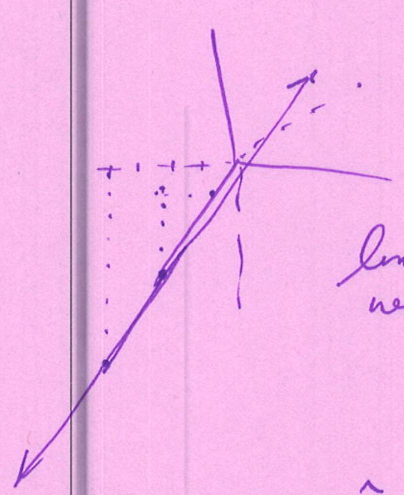
e. $\vec{r}(u,v) = u\hat{i} + v\hat{j} = \sqrt{uv}\,\hat{k}$

$z = \sqrt{uv} = \sqrt{xy}$

$z = \sqrt{xy}$

7. a. $\vec{r}(t) = t\hat{i} + (2t-4)\hat{j} + (3t-7)\hat{k}$

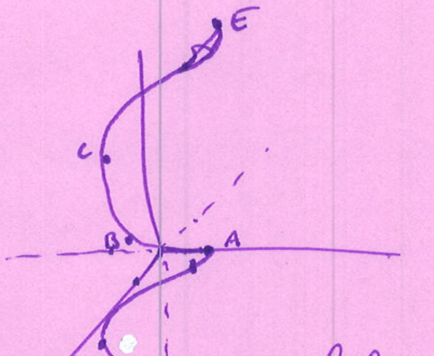
$(0,-4,-7)$
$(1,-2,-4)$



line needs only 2 points

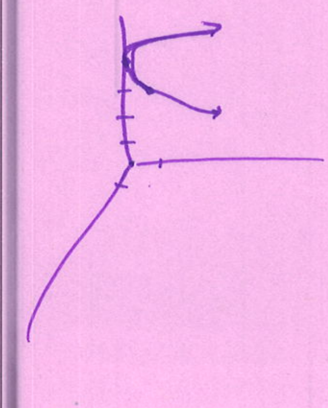
b. $\vec{r}(t) = \sin(t)\hat{i} + \cos(t)\hat{j} + t\hat{k}$

$t=0$ $(0,1,0)$
$t=\pi/2$ $(1,0,\pi/2)$
$t=\pi$ $(0,-1,\pi)$
$t=3\pi/2$ $(-1,0,3\pi/2)$
$t=2\pi$ $(0,1,2\pi)$
$t=-\pi/2$ $(-1,0,-\pi/2)$
$t=-\pi$ $(-1,0,-\pi)$
$t=-3\pi/2$ $(1,0,-3\pi/2)$
$t=-2\pi$ $(0,1,-2\pi)$



helix.

c. $\vec{r}(t) = t^2\hat{i} + t\hat{j} + 4\hat{k}$



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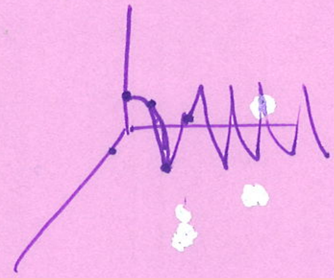
7d. $\vec{r}(t) = \sin^2(t)\,\hat{i} + t\,\hat{j} + \cos(t^2)\,\hat{k}$

t	x	y	z
0	0	0	1
$\pi/4$	$\frac{1}{2}$	$\pi/4$	.8157
$\pi/2$	1	$\pi/2$	-.7812
$3\pi/4$	$\frac{1}{2}$	$3\pi/4$	.744
π	0	π	-.902685



8\. a. $\lim\limits_{(x,y)\to(2,1)} \frac{x-y-1}{\sqrt{x-y}-1} \cdot \frac{\sqrt{x-y}+1}{\sqrt{x-y}+1} = \lim\limits_{(x,y)\to(2,1)} \frac{\cancel{(x-y-1)}(\sqrt{x-y}+1)}{\cancel{x-y-1}} = \lim\limits_{(x,y)\to(2,1)} \sqrt{x-y}+1 = 2$

b. $\lim\limits_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2} = \lim\limits_{r\to 0} \frac{\cancel{r^2}\cos\theta\sin\theta}{\cancel{r^2}} = \text{DNE}$

c. $\lim\limits_{(x,y)\to(0,0)} \frac{x^2-y^2}{\sqrt{x^2+y^2}} = \lim\limits_{r\to 0} \frac{r^2\cos^2\theta - r^2\sin^2\theta}{r} = \lim\limits_{r\to 0} r(\cos^2\theta - \sin^2\theta) = 0$

d. $\lim\limits_{(x,y)\to(0,0)} \frac{x^2+y^2}{xy} = \lim\limits_{r\to 0} \frac{\cancel{r^2}}{\cancel{r^2}\cos\theta\sin\theta} = \text{DNE}$

e. $\lim\limits_{(x,y)\to(0,0)} \frac{x^2y^2}{x^4 3y^4}$ let $y = kx$ $\lim\limits_{x\to 0} \frac{x^2\cdot k^2x^2}{x^4+3k^4x^4} = \lim\limits_{x\to 0} \frac{\cancel{x^4}(k^2)}{\cancel{x^4}(1+3k^4)}$

$= \frac{k^2}{1+3k^4}$ DNE

f. $\lim\limits_{(x,y)\to(0,0)} \frac{4x^2+y^2}{x^4+3y^4}$ let $y = kx$ $\lim\limits_{x\to 0} \frac{4x^2+k^2x^2}{x^4+3k^4y^4} = \lim\limits_{x\to 0} \frac{\cancel{x^2}(4+k^2)}{x^{\cancel{4}\,2}(1+3k^4)}$

$= \lim\limits_{x\to 0} \frac{4+k^2}{x^2(1+3k^4)} = \infty$ (all terms positive)

g. $\lim\limits_{(x,y)\to(0,0)} \frac{y^2\sin^2 y}{x^4+y^4}$ let $y = kx$ $\lim\limits_{x\to 0} \frac{k^2x^2\sin^2 x}{x^4+k^4x^4} = \lim\limits_{x\to 0} \frac{k^2\cancel{x^2}\sin^2 x}{x^{\cancel{4}\,2}(1+k^4)}$

$= \lim\limits_{x\to 0} \left(\frac{k^2}{1+k^4}\right)\underbrace{\left(\frac{\sin^2 x}{x^2}\right)}_{=1} = \frac{k^2}{1+k^4}$ DNE

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8h. $\lim_{(x,y)\to(0,0)} \dfrac{e^{-x^2-y^2}-1}{x^2+y^2} = \lim_{r\to 0} \dfrac{e^{-r^2}-1}{r^2}$ L'Hopitals

$\lim_{r\to 0} \dfrac{-\cancel{2r}e^{-r^2}}{\cancel{2r}} = \lim_{r\to 0} -e^{-r^2} = -1$

i. $\lim_{(x,y)\to(0,0)} \dfrac{x^2y^5}{2x^4+3y^{10}}$ let $x=ky^{5/2}$ $\lim_{y\to 0} \dfrac{k^2y^5 \cdot y^5}{2k^4y^{10}+3y^{10}} =$

$\lim_{y\to 0} \dfrac{k^2\cancel{y^{10}}}{\cancel{y^{10}}(2k^4+3)} = \dfrac{k^2}{(2k^4+3)}$ DNE

j. $\lim_{(x,y)\to(0,0)} \dfrac{3x^3+2\sqrt{y}}{x^2+y^2}$ let $y=kx$ $\lim_{x\to 0} \dfrac{3x^3+2\sqrt{kx}}{x^2+k^2x^2} =$

$\lim_{x\to 0} \dfrac{3x^{\cancel{3}}}{\cancel{x^2}(1+k^2)} + \lim_{x\to 0} \dfrac{2\sqrt{k}\,x^{1/2}}{x^{\cancel{2}\,3/2}(1+k^2)} = \lim_{x\to 0} \dfrac{3x}{1+k^2} + \lim_{x\to 0} \dfrac{2\sqrt{k}}{x^{3/2}(1+k^2)} = 0 + \text{DNE}$

k. $\lim_{(x,y,z)\to(0,0,0)} \arctan\left[\dfrac{1}{x^2+y^2+z^2}\right] = \lim_{\rho\to 0} \arctan\left[\dfrac{1}{\rho^2}\right] = \pi/2$ DNE

l. $\lim_{(x,y,z)\to(0,0,0)} \dfrac{xy+yz^2+xz^2}{x^2+y^2+z^2} = \lim_{\rho\to 0} \dfrac{\rho^2\sin^2\varphi\cos\theta\sin\theta + \rho^3\sin\varphi\sin\theta\cos^2\varphi + \rho^3\sin\varphi\cos\theta\cos^2\varphi}{\rho^2}$

$= \lim_{\rho\to 0} \sin^2\varphi\cos\theta\sin\theta + \rho\sin\varphi\sin\theta\cos^2\varphi + \rho\sin\varphi\cos\theta\cos^2\varphi = \text{DNE}$

m. $\lim_{(x,y)\to(0,0)} \dfrac{x-y}{\sqrt{x}-\sqrt{y}} \cdot \dfrac{\sqrt{x}+\sqrt{y}}{\sqrt{x}+\sqrt{y}} = \lim_{(x,y)\to(0,0)} \dfrac{\cancel{(x-y)}(\sqrt{x}+\sqrt{y})}{\cancel{x-y}} = 0$

n. $\lim_{(x,y)\to(0,0)} \dfrac{2x-y^2}{2x^2+y}$ let $y=kx^2$ $\lim_{x\to 0} \dfrac{2x-k^2x^4}{2x^2+k^2x^2} = \lim_{x\to 0} \dfrac{x(2-k^2x^3)}{x^2(2+k^2)}$

$= \lim_{x\to 0} \dfrac{2}{x(2+k^2)} - \lim_{x\to 0} \dfrac{k^2x^{\cancel{3}\,2}}{\cancel{x}(2+k^2)} \to 0 = \text{DNE}$

o. $\lim_{(x,y)\to(0,0)} \dfrac{\sin\sqrt{x^2+y^2}}{\sqrt{x^2+y^2}} = \lim_{r\to 0} \dfrac{\sin r}{r} = 1$

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8 p. $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2y}{x^3+y^2}$ let $y^2 = kx^3$, $y = \pm k x^{3/2}$ $\lim\limits_{x\to 0} \dfrac{x^2 \cdot kx^{3/2}}{x^3+k^2x^3} = \lim\limits_{x\to 0} \dfrac{\cancel{x^{3/2}}\,x^{1/2} \cdot k}{\cancel{x^3}(1+k^2)} = 0$

when x is >0

DNE

q. $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^3y}{x^6+y^2}$ let $y^2 = x^6$ $\Rightarrow y = kx^3$ $\lim\limits_{x\to 0} \dfrac{x^3 \cdot kx^3}{x^6+k^3x^6} = \lim\limits_{x\to 0} \dfrac{\cancel{x^6}(k)}{\cancel{x^6}(1+k^3)}$.

DNE

r. $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2y}{x^4+3y^2}$ let $y = kx^2$ $\lim\limits_{x\to 0} \dfrac{x^2 \cdot kx^2}{x^4+3k^2x^4} = \lim\limits_{x\to 0} \dfrac{\cancel{x^4}k}{\cancel{x^4}(1+3k^2)}$ DNE

s. $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2ye^y}{x^4+4y^2}$ let $y = kx^2$ $\lim\limits_{x\to 0} \dfrac{x^2 \cdot kx^2 e^{kx^2}}{x^4+4k^2x^4} = \lim\limits_{x\to 0} \dfrac{\cancel{x^4}(k e^{kx^2})}{\cancel{x^4}(1+4k^2)}$

DNE

t. $\lim\limits_{(x,y)\to(0,0)} (x^2+y^2)\ln(x^2+y^2) = \lim\limits_{r\to 0} r^2 \ln(r^2)$

$= \lim\limits_{r\to 0} 2r^2 \ln r = \lim\limits_{r\to 0} \dfrac{2\ln r}{r^{-2}}$ L'Hopital's $= \lim\limits_{r\to 0} \dfrac{\frac{\cancel{2}}{r}}{-\cancel{2}r^{-3}} = \lim\limits_{r\to 0} \dfrac{-r^3}{r} =$

$\lim\limits_{r\to 0} -r^2 = 0$

u. $\lim\limits_{(x,y)\to(0,0)} \dfrac{2x^3y^2}{x^6+y^4}$ let $y^4 = x^6$, $y^2 = x^3$, $y = \pm k x^{3/2}$ $\lim\limits_{x\to 0} \dfrac{2x^3k^2x^3}{x^6+k^4x^6} = \lim\limits_{x\to 0} \dfrac{\cancel{x^6}(2k^2)}{\cancel{x^6}(1+k^4)}$.

DNE

v. $\lim\limits_{(x,y,z)\to(0,0,0)} \dfrac{xyz}{x^2+y^2+z^2} = \lim\limits_{\rho\to 0} \dfrac{\rho^3 \cos\theta \sin\theta \sin^2\varphi \cos\varphi}{\cancel{\rho^2}} = 0$

w. $\lim\limits_{(x,y,z)\to(0,0,0)} \dfrac{xy+yz+xz}{x^2+y^2+z^2} = \lim\limits_{\rho\to 0} \dfrac{\cancel{\rho^2}\cos\theta\sin\theta\sin^2\varphi + \cancel{\rho^2}\sin\theta\sin\varphi\cos\varphi + \cancel{\rho^2}\cos\theta\sin\varphi\cos\varphi}{\cancel{\rho^2}}$

DNE