201 Homework #2 Key

1a. $\vec{u}\cdot\vec{v} = 5-6-12 = -13$

$$\vec{u}\times\vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & 4 \\ 5 & -2 & -3 \end{vmatrix} = (-9+8)\hat{i} - (-3-20)\hat{j} + (-2-15)\hat{k}$$
$$= -\hat{i} + 23\hat{j} - 17\hat{k}$$

b. $\vec{u}\cdot\vec{v} = 27+8-5 = 30$

$$\vec{u}\times\vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 9 & -4 & 1 \\ 3 & -2 & -5 \end{vmatrix} = (20+2)\hat{i} - (-45-3)\hat{j} + (-18+12)\hat{k}$$
$$22\hat{i} + 48\hat{j} - 6\hat{k}$$

2. $$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -8 & -6 & 4 \\ 10 & -12 & -2 \end{vmatrix} = (12+48)\hat{i} - (16-40)\hat{j} + (96+60)\hat{k}$$
$$60\hat{i} + 24\hat{j} + 156\hat{k} \quad \div 12$$
$$5\hat{i} + 2\hat{j} + 13\hat{k} \qquad \sqrt{25+4+169} = \sqrt{198} = 3\sqrt{22}$$

$$\frac{5}{3\sqrt{22}}\hat{i} + \frac{2}{3\sqrt{22}}\hat{j} + \frac{13}{3\sqrt{22}}\hat{k}$$

3. $\vec{u} = \langle 1, 2, 3\rangle$
$\vec{v} = \langle 5, 4, 1\rangle$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 5 & 4 & 1 \end{vmatrix} = (2-12)\hat{i} - (1-15)\hat{j} + (4-10)\hat{k}$$
$$-10\hat{i} + 14\hat{j} - 6\hat{k} \quad \div 2$$
$$-5\hat{i} + 7\hat{j} - 3\hat{k}$$

$$2\sqrt{25+49+9} = 2\sqrt{83}$$

4. $$\begin{vmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & -1 & 2 \end{vmatrix} = 2(4+1) - \cancel{1(0-0)}^{0} + 0\cancel{(0-0)}^{0} = 10$$

5.
a. $\vec{u} = \langle 1, -5, 4\rangle$

$x = 1t+2$
$y = -5t+4$ or $\vec{r}(t) = (t+2)\hat{i} + (4-5t)\hat{j} + (4t-3)\hat{k}$
$z = 4t-3$

b. $\vec{u} = \langle 11, -4, 1\rangle$ $\quad \dfrac{x+8}{11} = \dfrac{y-2}{-4} = \dfrac{z-4}{1}$

c. $\vec{u} = \langle 1, 2, 1\rangle$ $\quad \dfrac{x-1}{1} = \dfrac{y+1}{2} = \dfrac{z-1}{1}$

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6. $\vec{n} = \langle 2, -1, 3 \rangle$

a. $2(x-5) - (y+3) + 3(z+4) = 0$

b. $\vec{n} = \langle -2, 2, 0 \rangle$

$-2(x+6) + 2(y-0) + 0(z-8) = 0 \Rightarrow -2(x+6) + 2y = 0$

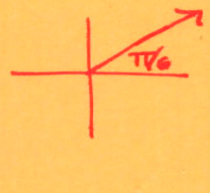
c. $\langle 1, 1, 4 \rangle \quad \langle -3, -4, 2 \rangle$

$\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 1 & 4 \\ -3 & -4 & 2 \end{vmatrix} = (2+16)\hat{\imath} - (2+12)\hat{\jmath} + (-4+3)\hat{k} = \langle 18, -14, -1 \rangle$

$18(x-2) - 14(y-3) - (z+2) = 0$

d. $\langle 0, 1, 0 \rangle \leftarrow$ y-axis

$\langle \frac{\sqrt{3}}{2}, 0, \frac{1}{2} \rangle$



$\cos \pi/6\, \hat{\imath} + \sin \pi/6\, \hat{k}$

$\vec{n} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & 1 & 0 \\ \sqrt{3}/2 & 0 & 1/2 \end{vmatrix} = \frac{1}{2}\hat{\imath} - 0\hat{\jmath} + -\frac{\sqrt{3}}{2}\hat{k} \quad \langle \frac{1}{2}, 0, -\frac{\sqrt{3}}{2} \rangle$

$\frac{1}{2}(x-0) + 0(y-0) - \frac{\sqrt{3}}{2}(z-0) = 0 \Rightarrow x - \sqrt{3}z = 0$

e. $\langle -2, 1, 1 \rangle, \langle -3, 4, -1 \rangle$

$\vec{n} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -2 & 1 & 1 \\ -3 & 4 & -1 \end{vmatrix} = (-1-4)\hat{\imath} - (2+3)\hat{\jmath} + (-8+3)\hat{k} =$

$-5\hat{\imath} - 5\hat{\jmath} - 5\hat{k} \quad \div 5$ or $(-5) \quad \langle 1, 1, 1 \rangle$

$(x-1) + (y-4) + z = 0$

f. $\langle -3, -1, -2 \rangle \quad \langle 2, -3, 1 \rangle$

$\vec{n} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -3 & -1 & -2 \\ 2 & -3 & 1 \end{vmatrix} = (-1-6)\hat{\imath} - (-3+4)\hat{\jmath} + (9+2)\hat{k}$

$-7\hat{\imath} - \hat{\jmath} + 11\hat{k}$

$-7(x-2) - (y-2) + 11(z-1) = 0$

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7. $3x+2y-7=z$

$x-4y+2(3x+2y-7)=0$

$x-\cancel{4y}+6x+\cancel{4y}-14=0$

$7x=14 \qquad x=2$

$6+2y-z=7$

$2y-z=1$

$2y-1=z$

$y=t, \quad z=2t-1$

$\vec{r}(t)=2\hat{\imath}+t\hat{\jmath}+(2t-1)\hat{k}$

$\langle 3,2,-1\rangle \cdot \langle 1,-4,2\rangle = \quad 3-8-2 = \dfrac{-7}{\sqrt{14}\sqrt{21}}=\cos\theta$

$\sqrt{9+4+1}=\sqrt{14} \qquad \sqrt{1+16+4}=\sqrt{21}$

$\theta \approx 65.91^\circ$ or 1.15 radians

8. a. $D=\dfrac{|\vec{PQ}\cdot\vec{n}|}{\|\vec{n}\|}$

$Q=(0,0,5) \qquad \vec{PQ}=\langle -2,-8,1\rangle$

$\vec{n}=\langle 2,1,1\rangle \qquad \vec{PQ}\cdot\vec{n}=-4-8+1=-11$

$\dfrac{|\vec{PQ}\cdot\vec{n}|}{\|\vec{n}\|}=\dfrac{|-11|}{\sqrt{4+1+1}}=\boxed{\dfrac{11}{\sqrt{6}}}$

b. $\vec{u}=\langle -1,1,-2\rangle \qquad Q=(1,2,0) \qquad \vec{PQ}=\langle 2,1,2\rangle$

$D=\dfrac{\|\vec{PQ}\times\vec{u}\|}{\|\vec{u}\|}$

$\vec{PQ}\times\vec{u}=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 2 & 1 & 2\\ -1 & 1 & -2\end{vmatrix}=(-2-2)\hat{\imath}-(-4+2)\hat{\jmath}+(2+1)\hat{k}$

$\langle -4,2,3\rangle$

$\|\vec{PQ}\times\vec{u}\|=\sqrt{16+4+9}=\sqrt{29}$

$D=\dfrac{\sqrt{29}}{\sqrt{1+1+4}}=\dfrac{\sqrt{29}}{\sqrt{6}}=\boxed{\sqrt{\dfrac{29}{6}}}$

9. a. $D: \{t \mid -2\le t\le 2\}$

$4-t^2\ge 0$

$4\ge t^2$

$\|\vec{r}(t)\|=\sqrt{(\sqrt{4-t^2})^2+(t^2)^2+(-6t)^2}=\sqrt{4-t^2+t^4+36t^2}=\sqrt{t^4+35t^2+4}$

b. $D: \{t \mid t>0\}$

$\|\vec{r}(t)\|=\sqrt{(\ln t-1)^2+t^2}=\sqrt{\ln^2 t-2\ln t+1+t^2}$

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9c. D: { t | any real # }

$\|\vec{r}(t)\| = \sqrt{9\cos^2 t + 4\sin^2 t + (t^4)^2} = \sqrt{4 + t^4 + 5\cos^2 t}$

$\underbrace{4\cos^2 t + 5\cos^2 t + 4\sin^2 t}_{=4}$

d. D: { t | $t \neq -1$ }

$\|\vec{r}(t)\| = \sqrt{(\sqrt[3]{t})^2 + \left(\frac{1}{t+1}\right)^2 + (t+2)^2} = \sqrt{\sqrt[3]{t^2} + \frac{1}{(t+1)^2} + t^2 + 4t + 4}$

e. D: { t | $t \geq 0$ }

$\|\vec{r}(t)\| = \sqrt{(1-t)^2 + (\sqrt{t})^2} = \sqrt{t^2 - 2t + 1 + t} = \sqrt{t^2 - t + 1}$

10. a. let $x = t$, $y = 4 - t$ $\qquad \vec{r}(t) = t\,\hat{i} + (4-t)\,\hat{j}$

(instead of t, you can choose any function of t) answers will vary

let $x = e^t$, $y = 4 - e^t$ $\qquad \vec{r}(t) = e^t\,\hat{i} + (4 - e^t)\,\hat{j}$

b. let $x = 5\cos t$, $y = 5\sin t$ $\qquad \vec{r}(t) = 5\cos t\,\hat{i} + 5\sin t\,\hat{j}$

another alternative is to switch signs or coordinates

$\vec{r}(t) = 5\sin t\,\hat{i} + 5\cos t\,\hat{j}$

c. let $t = x$ $\quad y = 4 - t^2$ $\qquad \vec{r}(t) = t\,\hat{i} + (4 - t^2)\,\hat{j}$

(or choose another function of t – answers will vary

$\vec{r}(t) = t^3\,\hat{i} + (4 - t^6)\,\hat{j}$

11. a. $f(x,y) = 4 - x^2 - y^2$ $\quad f(0,0) = 4$; $f(2,3) = 4 - 4 - 9 = -9$; $f(1,y) = 4 - 1 - y^2 = 3 - y^2$;

$f(x,0) = 4 - x^2$; $f(t, t^2) = 4 - t^2 - t^4$

b. $f(x,y,z) = \sqrt{x+y+z}$; $f(0,0,1) = \sqrt{1} = 1$; $f(2,3,9) = \sqrt{3+2+9} = \sqrt{14}$;

$f(1,y,0) = \sqrt{1+y}$; $f(x,0,x) = \sqrt{2x}$, $f(t,t^2,t^3) = \sqrt{t + t^2 + t^3}$

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11c. $f(x,y) = \arcsin(x+y)$; $f(0,0) = 0$; $f(\frac{1}{4},\frac{1}{4}) = \arcsin(\frac{1}{4}+\frac{1}{4}) = \frac{\pi}{6}$;

$f(1,y) = \arcsin(1+y)$; $f(x,0) = \arcsin(x)$; $f(t,t) = \arcsin(2t)$

d. $f(x,y) = \ln(xy-6)$; $f(5,e) = \ln(5e-6)$; $f(e,1) = \ln(e-6)$; $f(1,y)$

$\ln(y-6)$; $f(x,0) = \ln(-6)$ not defined; $f(t,e^t) = \ln(te^t-6)$.

12a. $x+y=0 \Rightarrow y=-t \qquad z = (t)^2 + (-t)^2 = 2t^2$

$x=t$

$\vec{r}(t) = t\hat{\imath} - t\hat{\jmath} + 2t^2\hat{k}$

b. $x = 1+\sin t \qquad x+z=2 \Rightarrow 1+\sin t + z = 2 \Rightarrow z = 1-\sin t$

$y^2 = 4-x^2-z^2 \Rightarrow y^2 = 4-(1+\sin t)^2-(1-\sin t)^2 =$

$4-(1+2\sin t+\sin^2 t)-(1-2\sin t+\sin^2 t) = 4-1-\cancel{2\sin t}-\sin^2 t-1+\cancel{2\sin t}-\sin^2 t$

$y^2 = 2-2\sin^2 t = 2(1-\sin^2 t) = 2\cos^2 t$

$y = \sqrt{2}\cos t$

$\vec{r}(t) = (1+\sin t)\hat{\imath} + \sqrt{2}\cos t\,\hat{\jmath} + (1-\sin t)\hat{k}$

c. $z=t \Rightarrow x=t^2 \qquad 4x^2+4y^2+z^2=16 \Rightarrow 4y^2 = 16-4x^2-z^2$

$\frac{4y^2}{4} = 16-4(t^2)^2-t^2 = \frac{16-4t^4-t^2}{4}$

$y^2 = 4-t^4-\frac{1}{4}t^2$

$y = \pm\sqrt{4-t^4-\frac{1}{4}t^2}$

half the curve is $\vec{r}_1(t) = t^2\hat{\imath} + \sqrt{4-t^4-\frac{1}{4}t^2}\,\hat{\jmath} + t\hat{k}$

other half is $\vec{r}_2(t) = t^2\hat{\imath} - \sqrt{4-t^4-\frac{1}{4}t^2}\,\hat{\jmath} + t\hat{k}$

13.a. $\vec{r}(t) = 3\cos t\,\hat{\imath} + 3\sin t\,\hat{\jmath}$ or

$y = \pm\sqrt{9-x^2}$

$\vec{r}_1(t) = (t+3)\hat{\imath} + \sqrt{9-t^2}\,\hat{\jmath}$

$\vec{r}_2(t) = (t-3)\hat{\imath} - \sqrt{9-t^2}\,\hat{\jmath}$

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13b.

$\vec{r}(t) = 4\cos t\,\hat{\imath} + 3\sin t\,\hat{\jmath} \qquad t \in [0, \pi/2]$

$\frac{x^2}{16} + \frac{y^2}{9} = 1 \Rightarrow 9x^2 + 16y^2 = 144 \Rightarrow y = \pm\sqrt{\frac{144 - 9x^2}{16}}$

$\vec{r}(t) = (4-t)\hat{\imath} + \frac{1}{4}\sqrt{144-9x^2}\,\hat{\jmath}$

c. $\vec{r}_1(t) = 5t\,\hat{\imath} + 4t\,\hat{\jmath}$

$\vec{r}_2(t) = 5\hat{\imath} - 4t\,\hat{\jmath}$

$\vec{r}_3(t) = (5-5t)\hat{\imath} + 0\hat{\jmath}$

you can replace t by a function of t as long as the range of the function includes [0,1].

d. $\vec{r}_1(t) = t\,\hat{\imath}$

$\vec{r}_2(t) = 1\hat{\imath} + t\hat{k}$

$\vec{r}_3(t) = 1\hat{\imath} + t\hat{\jmath} + 1\hat{k}$

ditto

e. $\vec{r}_1(t) = t\,\hat{\imath} + t^2\hat{\jmath} \qquad [0,2]$

$\vec{r}_2(t) = (2-t)\hat{\imath} + 4\hat{\jmath} \qquad [0,2]$

f. $\vec{r}(t) = 4\tan t\,\hat{\imath} + 2\sec t\,\hat{\jmath}$

14. a. i $\begin{bmatrix} 3 & 1 \\ -1 & 4 \end{bmatrix} + \begin{bmatrix} 9 & 3 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 12 & 4 \\ 0 & 4 \end{bmatrix}$

ii. $\begin{bmatrix} 18 & 6 \\ 2 & 0 \end{bmatrix} - \begin{bmatrix} 6 & -6 \\ -12 & 12 \end{bmatrix} = \begin{bmatrix} 12 & 12 \\ 14 & -12 \end{bmatrix}$

b. i $\begin{bmatrix} 12 & 4 \\ -4 & 16 \end{bmatrix}$ ii $\begin{bmatrix} -5 & 0 & 10 \end{bmatrix}$

c i. $\begin{bmatrix} 3 & 1 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} 9 & 3 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 27+1 & 9+0 \\ -9+4 & -3+0 \end{bmatrix} = \begin{bmatrix} 28 & 9 \\ -5 & -3 \end{bmatrix}$

ii. $\begin{bmatrix} 1 & 3 & 4 \\ -2 & 1 & 0 \\ 3 & -4 & 1 \end{bmatrix}\begin{bmatrix} 0 & -3 & 5 \\ 1 & -4 & 0 \\ -1 & 2 & -7 \end{bmatrix} = \begin{bmatrix} 0+3-4 & -3-12+8 & 5+0-28 \\ 0+1+0 & 6-4+0 & -10+0+0 \\ 0-4-1 & -9+16+2 & 15+0-7 \end{bmatrix} = \begin{bmatrix} -1 & -7 & -23 \\ 1 & 2 & -10 \\ -5 & 9 & 8 \end{bmatrix}$

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14 c iii. $\begin{bmatrix} 6 & -7 \\ 11 & -5 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 2 & -2 \\ -4 & 4 \end{bmatrix} = \begin{bmatrix} 12+28 & -12-28 \\ 22+20 & -22-20 \\ 4-12 & -4+12 \end{bmatrix} = \begin{bmatrix} 40 & -40 \\ 42 & -42 \\ -8 & 8 \end{bmatrix}$

iv. $\begin{bmatrix} 0 & -3 & 5 \\ 1 & -4 & 0 \\ -1 & 2 & -7 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \end{bmatrix}$ not defined

d. i $\begin{bmatrix} 3 & -1 \\ 1 & 4 \end{bmatrix}$ ii $\begin{bmatrix} 6 & 11 & 2 \\ -7 & -5 & 3 \end{bmatrix}$

e. i $\left| \begin{bmatrix} 2 & -2 \\ -4 & 4 \end{bmatrix} \right| = 8-8=0$

ii. $\begin{vmatrix} 1 & 3 & 4 \\ -2 & 1 & 0 \\ 3 & -4 & 1 \end{vmatrix} = 1\begin{vmatrix} 1 & 0 \\ -4 & 1 \end{vmatrix} - 3\begin{vmatrix} -2 & 0 \\ 3 & 1 \end{vmatrix} + 4\begin{vmatrix} -2 & 1 \\ 3 & -4 \end{vmatrix}$

$= 1(1) - 3(-2) + 4(8-3) = 1+6+20 = 27$

f. i. $(3-\lambda)(4-\lambda) - 6 = 0$

$\lambda^2 - 7\lambda + 12 - 6 = 0$

$\lambda^2 - 7\lambda + 6 = 0$

$(\lambda-1)(\lambda-6) = 0$

$\lambda = 1, 6$

$\begin{bmatrix} 3-1 & 2 \\ 3 & 4-1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 3 & 3 \end{bmatrix}$ $\dfrac{x_1 + x_2 = 0}{x_1 = -x_2}$ $x_2 = x_2$ $\vec{v}_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

$\begin{bmatrix} 3-6 & 2 \\ 3 & 4-6 \end{bmatrix} = \begin{bmatrix} -3 & 2 \\ 3 & -2 \end{bmatrix}$ $\dfrac{-3x_1 + 2x_2 = 0}{x_1 = \frac{2}{3}x_2}$ $x_2 = x_2$ $\vec{v}_2 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

ii. $\begin{bmatrix} 3-\lambda & 1 & 2 \\ 0 & -1-\lambda & 0 \\ 0 & 6 & 4-\lambda \end{bmatrix} = (3-\lambda)\begin{vmatrix} -1-\lambda & 0 \\ 6 & 4-\lambda \end{vmatrix} = (3-\lambda)\left[(-1-\lambda)(4-\lambda)\right] - 0 = 0$

$\lambda = 3, -1, 4$

$\lambda = 3$ $\begin{bmatrix} 0 & 1 & 2 \\ 0 & -4 & 0 \\ 0 & 6 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ $\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ $x_1 = x_1$, $x_2 = 0$, $x_3 = 0$

$\lambda = -1$ $\begin{bmatrix} 4 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 6 & 5 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 7/24 \\ 0 & 1 & 5/6 \\ 0 & 0 & 0 \end{bmatrix}$ $\vec{v}_2 = \begin{bmatrix} -7 \\ -20 \\ 24 \end{bmatrix}$

$\lambda = 4$ $\begin{bmatrix} 1 & 1 & 2 \\ 0 & -5 & 0 \\ 0 & 6 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $x_2 = 0$ $\vec{v}_3 = \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$