MAT 192, Series Activity 3, Fall 2016

Group Members KEY

Instructions: In groups of 2-4 students, discuss each of the problems below. Discuss solution strategies, appropriate notation, etc. Then each student should write up solutions. Justify all steps. Your group may submit one copy of the assignment with the name of all group members for grading.

1. Determine if the series converges. State the test you are using and show all the required elements of the test to justify your conclusion.

a. $\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{3}}+\frac{1}{2}-\frac{1}{\sqrt{5}}+\frac{1}{\sqrt{6}}-\cdots$ $\sum_{n=2}^{\infty}\frac{(-1)^n}{\sqrt{n}}$

Converges by alternating series test

b. $\sum_{n=1}^{\infty}\frac{(-1)^n(3n-1)}{2n+1}$

diverges by alt. series test.

$\lim_{n\to\infty}\frac{3n-1}{2n+1}=\frac{3}{2}\neq 0$

h. $\sum_{n=1}^{\infty}(-1)^{n-1}e^{2/n}$

$\lim_{n\to\infty} e^{2/n}=1\neq 0$ diverges by alt./nth term test.

c. $\sum_{n=0}^{\infty}\frac{\sin\left[\left(n+\frac{1}{2}\right)\pi\right]}{1+\sqrt{n}}=\sum_{n=0}^{\infty}\frac{(-1)^n}{1+\sqrt{n}}$

Converges by alt. series test

i. $\sum_{n=1}^{\infty}\frac{(-1)^n n^n}{n!}$ diverges by alt / nth term test

d. $\sum_{n=1}^{\infty}\frac{n}{8^n}$

Converges

$\lim_{n\to\infty}\frac{n+1}{8^{n+1}}\cdot\frac{8^n}{n}=\frac{1}{8}<1$

by ratio test

j. $\sum_{n=0}^{\infty}\frac{\left(\frac{1}{5}\right)^n}{n}$

$\lim_{n\to\infty}\sqrt[n]{\frac{(1/5)^n}{n}}=\frac{1/5}{1}=1/5<1$

Converges by root test

e. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^p}$ (for what values of p?

converges $p>0$

diverges for $p \leq 0$

$\lim_{n\to\infty} \frac{1}{n^p} = \begin{cases} \infty & p<0 \\ 1 & p=0 \\ 0 & p>0 \end{cases}$

f. $\sum_{n=1}^{\infty} \frac{n^{10}}{(-10)^{n+1}}$ ratio test

$\lim_{n\to\infty} \left| \frac{(n+1)^{10}}{(-10)^{n+2}} \cdot \frac{(-10)^{n+1}}{n^{10}} \right| = \frac{1}{10} < 1$

Converges

k. $\sum_{n=1}^{\infty} \frac{n^{100}100^n}{n!}$

$\lim_{n\to\infty} \frac{(n+1)^{100} 100^{n+1}}{(n+1)!} \cdot \frac{n!}{n^{100} 100^n} = 0 < 1$

Converges

g. $\sum_{n=1}^{\infty} \frac{3-\cos n}{n^{2/3}-2}$ $\leq \sum \frac{4}{n^{2/3}-2}$

diverges by limit comparison w/ $\frac{1}{n^{2/3}}$ (p-series $p \leq 1$)

l. $\sum_{n=1}^{\infty} \left(\frac{n^2+1}{2n^n+1}\right)^n$

$\lim_{n\to\infty} \sqrt[n]{\left(\frac{n^2+1}{2n^n+1}\right)^n} = \lim_{n\to\infty} \frac{n^2+1}{2n^n+1} = 0 < 1$

Converges by root test

2. Estimate the error on the series when $n = 50$.

a. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^6}$ alternating

$R \leq 51^{st}$ term $\left|\frac{(-1)}{51^6}\right| \approx 5.68 \times 10^{-11}$

e. $\sum_{n=1}^{\infty} (-1)^{n-1} n^2 e^{-n}$ alternating

$R \leq 51^{st}$ term $\leq$

$|51^2/e^{51}| \approx 1.85 \times 10^{-19}$

b. $\sum_{n=1}^{\infty} \frac{n}{n^4+1} < \sum \frac{1}{n^3}$

Converges by direct comparison w/ p-series

$\int_{51}^{\infty} \frac{n}{n^4+1}\,dn$ $u = n^2$, $du = 2n$ $= \frac{1}{2}\int_{2601}^{\infty} \frac{du}{u^2+1} =$

$\frac{1}{2}\left[\pi/2 - \arctan(2601)\right] \approx 1.92 \times 10^{-4}$

c. $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{\sqrt{n}} = \sum \frac{(-1)^n}{\sqrt{n}}$

Converges by alternating series

$R \le 51^{st}$ term $= \left|\frac{1}{\sqrt{51}}\right| \approx .14$

d. $\sum_{n=0}^{\infty} \frac{1}{n^2+4n+3} = \sum_{n=0}^{\infty} \frac{1}{(n+3)(n+1)}$

$\frac{A}{n+3} + \frac{B}{n+1} = \frac{An + A + Bn + 3B}{\sim} = \frac{1}{\sim}$

$A + B = 0$
$-A + 3B = 1$
$2B = -1$
$B = -1/2$
$A = 1/2$

$-\frac{1}{2}\sum\left(\frac{1}{n+1} - \frac{1}{n+3}\right)$ telescoping

exact sum

$-\frac{1}{2}\left(1 + \frac{1}{2}\right) = -\frac{3}{4}$

f. $\sum_{n=2}^{\infty} \frac{1}{n \ln^2 n}$ Converges by integral test (p-series)

$u = \ln n$
$du = \frac{1}{n}$

$\int_{53}^{\infty} \frac{1}{n \ln^2 n}\,dn = \int_{3.97}^{\infty} \frac{1}{u^2}\,du$

$= \frac{-1}{u}\Big|_{3.97}^{0} \approx .25187$

g. $\sum_{n=1}^{\infty} \frac{\cos^2 \pi n}{n^2+1} = \sum \frac{1}{n^2+1}$

Converges by integral test

$\int_{51}^{\infty} \frac{1}{n^2+1}\,dn = \arctan n \Big|_{51}^{\infty}$

$\pi/2 - \arctan 51 \approx .0196$

h. $\sum_{n=2}^{\infty} \frac{1}{n^2-1} = \sum\left(\frac{1}{n-1} - \frac{1}{n+1}\right)$

telescoping

exact sum is $1 + \frac{1}{2} = \frac{3}{2}$