MAT 192, Series Activity 2, Fall 2016

Group Members KEY

Instructions: In groups of 2-4 students, discuss each of the problems below. Discuss solution strategies, appropriate notation, etc. Then each student should write up solutions. Justify all steps. Your group may submit one copy of the assignment with the name of all group members for grading.

1. Use a comparison test to determine the convergence of the series. For each problem, you will need to state all of the following:
   i. Which comparison test you are using
   ii. State the series you are comparing the problem to
   iii. Check the conditions for the comparison that the series satisfies the comparison conditions
   iv. Check the convergence or divergence of the comparison series: which test are you using to determine convergence of the comparison series; show work to prove your convergence/divergence claim
   v. Conduct your comparison and clearly state the results
   vi. If your test proves inconclusive, repeat the above steps with another test or series

a. $\sum_{n=1}^{\infty} \frac{n}{2n^3+1}$ $\leq \sum \frac{1}{2n^2}$

Converges by direct comparison w/ p-series.

b. $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{3n^4+1}}$ $\leq \sum \frac{1}{\sqrt[3]{3}\, n^{4/3}}$

Converges by direct comparison w/ p-series

c. $\sum_{k=1}^{\infty} \frac{k \sin^2 k}{1+k^3}$ $\leq \sum \frac{k}{1+k^3} \leq \sum \frac{1}{k^2}$

Converges by direct comparison

d. $\sum_{n=1}^{\infty} 5^{-n} \cos^2 n$ $\leq \sum \left(\frac{1}{5}\right)^n$

Converges by direct comparison

e. $\sum_{n=1}^{\infty} \frac{n-1}{n^2\sqrt{n}}$ direct comparison

$\sum \frac{1}{n^{3/2}}$ converges by p-series $p > 1$

f. $\sum_{n=1}^{\infty} \frac{1+4^n}{1+3^n}$ limit comparison w/ $\sum \frac{4^n}{3^n}$

diverges

g. $\sum_{n=1}^{\infty} \frac{n!}{n^n}$

$\lim_{n\to\infty} \frac{(n+1)!}{(n+1)^{n+1}} \cdot \frac{n^n}{n!} =$

$\lim_{n\to\infty} \left(\frac{n}{n+1}\right)^n = \lim_{n\to\infty} \left(1-\frac{1}{n+1}\right)^n = \frac{1}{e}$

(use L'Hôpital's to prove this, after applying logs)