MAT 192, Series Activity 1, Fall 2016

Group Members KEY

Instructions: In groups of 2-4 students, discuss each of the problems below. Discuss solution strategies, appropriate notation, etc. Then each student should write up solutions. Justify all steps. Your group may submit one copy of the assignment with the name of all group members for grading.

1. Write an expression for the series $-3+2-\frac{4}{3}+\frac{8}{9}-\frac{16}{27}+\cdots$ in summation notation, and then calculate the sum (if one exists). What kind of series is it?

$$-3\left(1-\tfrac{2}{3}+\frac{4}{9}-\frac{8}{27}+\frac{16}{81}-\cdots\right)=\sum_{n=0}^{\infty}-3\left(-\tfrac{2}{3}\right)^n$$

2. Write an expression for the sequence $\frac{1}{6},\frac{2}{12},\frac{3}{20},\frac{4}{30},\frac{5}{42},\ldots$ in terms of n.

$$a_n=\left\{\frac{n}{(n+1)(n+2)}\right\}_{n=1}$$

3. Write an expression for the ~~series~~ sequence $1,\frac{2}{2},\frac{4}{6},\frac{8}{24},\frac{16}{120},\frac{32}{720},\ldots$ in ~~summation notation~~ terms of n.

$$a_n=\left\{\frac{2^n}{(n+1)!}\right\}_{n=0}$$

4. Write $0.\overline{46}$ and $0.2\overline{15}$ as a series. Use that to write each expression as the ratio of integers.

$$\sum_{n=0}^{\infty}\frac{46}{100}\left(\frac{1}{100}\right)^n=\frac{46/100}{1-1/100}=\frac{46}{\cancel{100}}\cdot\frac{\cancel{100}}{99}=\frac{46}{99}$$

$$\frac{2}{10}+\frac{15}{1000}\sum_{n=0}^{\infty}\left(\frac{1}{100}\right)^n=\frac{2}{10}+\frac{15/1000}{1-1/100}=\frac{15}{10\cancel{00}}\cdot\frac{\cancel{100}}{99}+\frac{2}{10}=\frac{2}{10}+\frac{15}{990}$$

$$=\frac{71}{330}$$

5. For each series below, state which test (or tests) that is (are) appropriate to determine if the series convergence diverges. Explain your reasoning. Then determine if the series converges, and if the series is geometric or telescoping, find the sum.

a. $\sum_{n=1}^{\infty} \frac{\sqrt{n}+4}{n^2}$ $= \sum \frac{1}{n^{3/2}} + \sum \frac{4}{n^2}$

both converge by p-series test

f. $\sum_{n=1}^{\infty} \frac{2}{n^2+2n}$ $\leq \sum \frac{2}{n^2}$

converges by direct comparison w/ pseries

b. $\sum_{n=2}^{\infty} \frac{1}{n \ln^3 n}$

converges by p-series test

integrate $\int_2^{\infty} \frac{1}{n \ln^3 n} dn =$ $u = \ln n$ $du = 1/n$ $\int \frac{1}{u^3} du$

g. $\sum_{n=1}^{\infty} \frac{3n-1}{2n+1}$ diverges by nth term test

$\lim_{n\to\infty} = \frac{3}{2}$

c. $\sum_{n=2}^{\infty} \frac{\ln n}{n^2}$

integral test $\int_2^{\infty} \frac{\ln n}{n^2} dn =$ $\lim_{n\to\infty}$ $-\frac{\ln n}{n} - \int -\frac{1}{n^2} dn$ converges

$u = \ln n$ $dv = n^{-2}$
$du = \frac{1}{n}$ $v = -\frac{1}{n}$

d. $\sum_{n=1}^{\infty} \frac{1}{n^2+6n+13}$ converges by direct comparison w/ $\sum \frac{1}{n^2}$ (p series)

(or use integral test)

e. $\sum_{n=3}^{\infty} \frac{n^2}{e^n}$

integral test

$\int_3^{\infty} \frac{n^2}{e^n} dn =$ $u = n^2$ $dv = e^{-n}$ $du = 2n$ $v = -e^{-n}$

$-n^2 e^{-n} + \int_3^{\infty} + 2ne^{-n} dn = -n^2e^{-n} \mp 2ne^{-n} - 2e^{-n}$

$u = 2n$ $dv = e^{-n}$
$du = 2$ $v = -e^{-n}$

$\lim_{n\to\infty}$ converges