MAT 192, Quiz #4A, Fall 2016 Name KEY

Instructions: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Use integration by tables to integrate. State the formula number used.
   a. $\int x^2 \operatorname{csch}(x^3+1)\,dx$

$u = x^3+1$
$du = 3x^2\,dx$
$\frac{1}{3}du = x^2\,dx$

$\frac{1}{3}\int \operatorname{csch} u\,du$

formula 6.6

$\int \operatorname{csch}(ax)\,dx = \frac{1}{a}\ln\left|\frac{e^{ax}-1}{e^{ax}+1}\right| + C$

$\Rightarrow \frac{1}{3}\ln\left|\frac{e^u-1}{e^u+1}\right| + C = \boxed{\frac{1}{3}\ln\left|\frac{e^{x^3+1}-1}{e^{x^3+1}+1}\right| + C}$

   b. $\int \frac{\sec^2\theta\tan^2\theta}{\sqrt{9-\tan^2\theta}}\,d\theta$

$u = \tan\theta$
$du = \sec^2\theta$

$\int \frac{u^2}{\sqrt{9-u^2}}\,du$

formula 3.40

$\int \frac{x^2}{\sqrt{a^2-x^2}}\,dx = -\frac{x}{2}\sqrt{a^2-x^2} - \frac{a^2}{2}\arcsin\left(\frac{x}{a}\right) + C$

$-\frac{u}{2}\sqrt{9-u^2} - \frac{9}{2}\arcsin\left(\frac{u}{3}\right) + C$

$= \boxed{-\frac{\tan\theta}{2}\sqrt{9-\tan^2\theta} - \frac{9}{2}\arcsin\left(\frac{\tan\theta}{3}\right) + C}$

MAT 192, Quiz #4B, Fall 2016 Name KEY

Instructions: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Use Simpson's Rule with $n = 4$ to estimate $\int_0^4 \cos\sqrt{x}\,dx$.

$\Delta x = 1$

$f(x) \approx \frac{4}{3(4)}\left[f(0)+4f(1)+2f(2)+4f(3)+f(4)\right]$

$\frac{1}{3}\left[\cos 0 + 4\cos(1) + 2\cos\sqrt{2} + 4\cos\sqrt{3} + \cos 2\right] =$

$\approx .8049$

2. Determine if the integral $\int_1^\infty \frac{e^{-\sqrt{x}}}{\sqrt{x}}\,dx$ converges or diverges. If it converges, find the value.

$\int_1^\infty 2e^{-u}\,du$

$u = \sqrt{x} = x^{1/2}$

$du = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}\,dx$

$2du = \frac{1}{\sqrt{x}}\,dx$

$u = 1$ when $x = 1$

$u = \infty$ when $x = \infty$

$= \lim_{b\to\infty} \int_1^b 2e^{-u}\,du =$

$\lim_{b\to\infty} -2e^{-u}\Big|_1^b =$

$\lim_{b\to\infty} -2e^{-b} + 2e^{-1} = 0 + \frac{2}{e} = \boxed{\frac{2}{e}}$

Converges