MAT 192, Quiz #3, Fall 2016 Name KEY

Instructions: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Integrate $\int \frac{\sqrt{1+x^2}}{x^2}\,dx$ by trig substitution.

$x = \tan\theta$

$1+x^2 = 1+\tan^2\theta = \sec^2\theta$

$\sqrt{1+x^2} = \sqrt{\sec^2\theta} = \sec\theta$

$\sec^2\theta\, d\theta = dx$

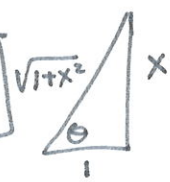
$$\int \frac{\sec\theta \cdot \sec^2\theta}{\tan^2\theta}\, d\theta$$

$$\int \frac{1}{\cos^3\theta}\cdot\frac{\cancel{\cos^2\theta}}{\sin^2\theta}\,d\theta = \int \frac{1}{\cos\theta\sin\theta}\,d\theta = \int \frac{2}{\sin 2\theta}\,d\theta = 2\int \csc 2\theta\, d\theta$$

$2\cos\theta\sin\theta = \sin 2\theta$

$$-\frac{2}{2}\ln|\csc 2\theta + \cot 2\theta| + C = -\ln\left|\frac{1}{2\sin\theta\cos\theta} + \frac{1-\tan^2\theta}{2\tan\theta}\right| + C$$

$$-\ln\left|\frac{1}{2}\frac{\sqrt{1+x^2}}{x}\cdot\frac{\sqrt{1+x^2}}{1} + \frac{1-x^2}{2x}\right| + C = \boxed{-\ln\left[\frac{1}{2}\left(\frac{1+x^2}{x} + \frac{1-x^2}{x}\right)\right] + C}$$



2. Use partial fractions to integrate $\int \frac{10}{(x-1)(x^2+9)}\,dx$.

$$\int \frac{A}{x-1} + \frac{Bx+C}{x^2+9}\,dx$$

$Ax^2+9 + Bx^2 - Bx + Cx - C = 10$

$A+B=0 \Rightarrow A=-B \leftarrow A=1$

$C-B=0 \quad C=B \leftarrow B=-1$

$9-C=10$

$-9 \quad -9$

$-C=1$

$C=-1$

$$\int \frac{1}{x-1} - \frac{x+1}{x^2+9}\,dx$$

$$\int \frac{1}{x-1} - \frac{x}{x^2+9} - \frac{1}{x^2+9}\,dx$$

$$\ln|x-1| - \frac{1}{2}\ln|x^2+9| - \frac{1}{3}\arctan\left(\frac{x}{3}\right) + C$$

3. For each integral below, determine which integration strategy is appropriate. You do not need to integrate.

a. $\int_{-1}^{1} \frac{e^{\arctan y}}{y^2+1} dy$

Substitution
$u = \arctan y$

d. $\int_0^{\pi} t \cos^2 t \, dt$

by parts
+ identity

b. $\int \frac{dx}{(1-x^2)^{3/2}}$

trig substitution

e. $\int \frac{3x-2}{x^2-2x-8} dx$

partial fractions

c. $\int \csc x \, dx$

basic formula