MAT 192, Quiz #11, Fall 2016 Name KEY

Instructions: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Find the first three non-zero terms of the Maclaurin series of the function $f(x) = \sinh x$. Sketch the graph of these terms compared to $f(x)$.

$c = 0$

n	n!	$f^{(n)}(x)$	$f^{(n)}(c)$	$(x-c)^n$	$\frac{f^{(n)}(c)}{n!}(x-c)^n$
0	1	$\sinh x$	0	1	$\frac{0}{1}(1) = 0$
1	1	$\cosh x$	1	(x)	$\frac{1}{1}x = x$
2	2	$\sinh x$	0	x^2	$\frac{0}{2}x^2 = 0$
3	6	$\cosh x$	1	x^3	$\frac{1}{6}x^3$
4	24	$\sinh x$	0	x^4	0
5	120	$\cosh x$	1	x^5	$\frac{1}{120}x^5$
6	720	$\sinh x$	0	x^6	0

$P_n(x) = x + \frac{1}{6}x^3 + \frac{1}{120}x^5 + \cdots$

2. If $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ and $\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$, find the first 5 terms of the Taylor series for $f(x) = e^{-x}\sin x$.

$$\left(1 - x + \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24} - \cdots\right)\left(x - \frac{1}{6}x^3 + \frac{1}{120}x^5 - \frac{1}{5040}x^7 \cdots\right)$$

$$x - \frac{1}{6}x^3 + \frac{1}{120}x^5 + \cdots - x^2 + \cancel{\frac{1}{6}x^4} + \cdots + \frac{x^3}{2} + -\frac{1}{12}x^5 + \cdots - \cancel{\frac{1}{6}x^4} + \cdots + \frac{1}{24}x^5 + \cdots$$

$$-\frac{1}{120}x^6 \qquad +\frac{1}{36}x^6$$

$$= x - x^2 + \frac{1}{3}x^3 - \frac{1}{30}x^5 + \frac{7}{360}x^6 + \cdots$$