MAT 192, Quiz #10, Fall 2016 Name KEY

Instructions: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Determine if the series $\sum_{n=1}^{\infty} \frac{(-1)^n 2^n n!}{(n+2)!}$ Converges conditionally or absolutely.

$$\lim_{n\to\infty} \left| \frac{2^{n+1}(n+1)!}{(n+3)!} \frac{(n+2)!}{2^n n!} \right| = \lim_{n\to\infty} \left| \frac{2^n \cdot 2\, n!(n+1)(n+2)!}{2^n \cdot n!(n+2)!(n+3)} \right| =$$

$$\lim_{n\to\infty} \frac{2(n+1)}{n+3} = 2 \quad \text{diverges}$$

2. Find the interval of convergence for the power series $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{\sqrt[3]{n}}$. Be sure to check the endpoints.

$$\lim_{n\to\infty} \left| \frac{x^{n+1}}{\sqrt[3]{n+1}} \frac{\sqrt[3]{n}}{x^n} \right| = \lim_{n\to\infty} x < 1$$

$(-1, 1)$

$\sum \frac{1}{\sqrt[3]{n}}$ diverges (p-series)

$\sum \frac{(-1)^n}{\sqrt[3]{n}}$ converges conditionally

$(-1, 1]$

3. Write the function $f(x) = \frac{5}{1-4x^2}$ as a power series.

$\sum_{n=0}^{\infty} ar^n$

$r = 4x^2$

$a = 5$

$$f(x) = \sum_{n=0}^{\infty} 5(4x^2)^n = \sum_{n=0}^{\infty} 5 \cdot 4^n \cdot x^{2n}$$