192 Homework #3 Key

1a.i. $\int_0^8 \sqrt[3]{x}\,dx, n=8 \quad \Delta x = \frac{8-0}{8} = 1$

$\approx \frac{1}{2}\left[\cancel{\sqrt[3]{0}}^{\,0} + 2\sqrt[3]{1} + 2\sqrt[3]{2} + 2\sqrt[3]{3} + 2\sqrt[3]{4} + 2\sqrt[3]{5} + 2\sqrt[3]{6} + 2\sqrt[3]{7} + \sqrt[3]{8}\right] = 11.73$

$(11.72959939\ldots)$

c. $\int_0^8 x^{1/3}\,dx = \frac{3}{4}x^{4/3}\Big|_0^8 = \frac{3}{4}(8)^{4/3} = \frac{3}{4}(2)^4 = 12$ (true)

b. $\approx \frac{1}{3}\left[\cancel{\sqrt[3]{0}}^{\,0} + 4\sqrt[3]{1} + 2\sqrt[3]{2} + 4\sqrt[3]{3} + 2\sqrt[3]{4} + 4\sqrt[3]{5} + 2\sqrt[3]{6} + 4\sqrt[3]{7} + \sqrt[3]{8}\right] = 11.86$

$(11.86317073\ldots)$

a. ii. $\int_1^4 \sqrt{\ln x}\,dx, n=6 \quad \Delta x = \frac{4-1}{6} = \frac{3}{6} = \frac{1}{2}$

$\approx \frac{1}{4}\left[\cancel{\sqrt{\ln 1}}^{\,0} + 2\sqrt{\ln 3/2} + 2\sqrt{\ln 2} + 2\sqrt{\ln 5/2} + 2\sqrt{\ln 3} + 2\sqrt{\ln 7/2} + \sqrt{\ln 4}\right] =$

$= 2.591333912$

b. $\approx \frac{1}{6}\left[\cancel{\sqrt{\ln 1}}^{\,0} + 4\sqrt{\ln 3/2} + 2\sqrt{\ln 2} + 4\sqrt{\ln 5/2} + 2\sqrt{\ln 3} + 4\sqrt{\ln 7/2} + \sqrt{\ln 4}\right] = 2.631976317$

c. by calc: 2.661415813

iii. a. $\int_{-1}^1 e^{e^x}\,dx, n=10 \quad \Delta x = \frac{1-(-1)}{10} = \frac{1}{5}$

$\approx \frac{1}{10}\left[e^{e^{-1}} + 2e^{e^{-4/5}} + 2e^{e^{-3/5}} + 2e^{e^{-2/5}} + 2e^{e^{-1/5}} + 2e^{e^0} + 2e^{e^{1/5}} + 2e^{e^{2/5}} + 2e^{e^{3/5}} + 2e^{e^{4/5}} + e^{e^1}\right]$

$= 8.363852972$

b. $\approx \frac{1}{15}\left[e^{e^{-1}} + 4e^{e^{-4/5}} + 2e^{e^{-3/5}} + 4e^{e^{-2/5}} + 2e^{e^{-1/5}} + 4e^{e^0} + 2e^{e^{1/5}} + 4e^{e^{2/5}} + 2e^{e^{3/5}} + 4e^{e^{4/5}} + e^{e^1}\right]$

$= 8.23511448$

c. by calculator 8.229784162

2. $E = 10^{-6}$

a. i. $\int_1^3 \frac{1}{\sqrt{x}}\,dx \quad E \approx \frac{\frac{3}{4}(3-1)^3}{12n^2} = 10^{-6}$

$f(x) = x^{-1/2} \quad f'(x) = -\frac{1}{2}x^{-3/2}$

$f''(x) = +\frac{3}{4}x^{-5/2} \quad f'''(x) = \frac{15}{8}x^{-7/2}$

$f^{iv}(x) = \frac{105}{16}x^{-9/2}$

$n^2 \geq \frac{3/4 \cdot 2^3}{12} 10^6$

$n \geq 707.106\ldots \quad \boxed{708}$

b. $10^{-6} = \frac{\frac{105}{16}(3-1)^5}{180n^4} \Rightarrow n^4 \geq \frac{\frac{105}{16}(2)^5 \cdot 10^6}{180} \quad n \geq 32.865 \Rightarrow \boxed{34}$

must be even for Simpson's Rule

192 Homework #3 Key

ii a. $\int_{-1}^{1}\sqrt{4-x^3}\,dx$

$f(x) = (4-x^3)^{1/2}$ $\quad f'(x) = \frac{1}{2}(4-x^3)^{-1/2}\cdot 3x^2 = \frac{3}{2}x^2(4-x^3)^{-1/2}$

$f''(x) = \frac{3x(x^3-16)}{4(4-x^3)^{3/2}}$

$f'''(x) = \frac{3(x^6-80x^3-128)}{8(4-x^3)^{5/2}}$

$f^{iv}(x) = \frac{9x^2(x^6-224x^3-1280)}{16(4-x^3)^{7/2}}$

$10^{-6} = \left|\frac{3(+1)(1-16)}{4(4-(+1))^{3/2}}\right| \cdot \frac{(1-(-1))^3}{12n^2}$

$n^2 \geq \frac{2.1650649\, 2^3}{12} 10^6$ $\quad n \geq 1201.4058\ldots$ $\quad \boxed{n=1202}$

b. $10^{-6} = \left|\frac{9(1)^2(1-224(1)-1280)}{16(4-1^3)^{7/2}}\right| \cdot \frac{(1-(-1))^5}{180n^4}$

$n^4 \geq \frac{18.07828 \cdot 2^5}{180} \cdot 10^6$ $\quad n \geq 42.3407\ldots$ $\quad \boxed{n=44}$ Simpson's n must be even

iii. $\int_1^2 e^{1/x}\,dx$

$f(x) = e^{1/x},\ f'(x) = \frac{-e^{1/x}}{x^2},\ f''(x) = \frac{e^{1/x}(2x+1)}{x^4}$

$f'''(x) = \frac{-e^{1/x}(6x^2+6x+1)}{x^6}$ $\quad f^{iv}(x) = \frac{e^{1/x}(24x^3+36x^2+12x+1)}{x^8}$

a. $10^{-6} = \frac{e^1(2+1)}{1} \cdot \frac{(2-1)^3}{12n^2}$

$n^2 \geq \frac{3e}{12} \cdot 10^6$ $\quad n \geq 824.36\ldots$ $\quad \boxed{n=825}$

b. $10^{-6} = \frac{e^1(24+36+12+1)}{1} \cdot \frac{(2-1)^3}{180n^4}$

Simpson's rule n must be even

$n^4 \geq \frac{73e}{180} 10^6$

$n \geq 32.403\ldots$ $\quad \boxed{n=34}$

3a. $\lim_{x\to 0} \frac{\arcsin x}{x}$ (this is $\frac{0}{0}$) $= \lim_{x\to 0} \frac{1/\sqrt{1-x^2}}{1} = 1$

b. $\lim_{x\to 0} \left(1+\frac{1}{x}\right)^{1/x} = L$ $\quad \lim_{x\to 0} \ln\left(1+\frac{1}{x}\right)^{1/x} = \ln L = \lim_{x\to 0} \frac{1}{x}\ln\left(1+\frac{1}{x}\right) =$

$\lim_{x\to 0} \frac{\ln(1+\frac{1}{x})}{x} = \frac{\infty}{0} =$ DNE (∞ on $+$, $-\infty$ on $-$) so L is DNE

L'Hopital's does not apply

c. $\lim_{x\to\infty} \frac{x^2}{\sqrt{x^2+1}}$ $\left(\frac{\infty}{\infty}\right) = \lim_{x\to\infty} \frac{2x}{\frac{1}{2}\cdot 2x(x^2+1)^{-1/2}} = \lim_{x\to\infty} 2\sqrt{x^2+1} = \infty$

d. $\lim_{x\to 0^+} x^3\cot x = \lim_{x\to 0^+} \frac{x^3}{\tan x}$ $\left(\frac{0}{0}\right) = \lim_{x\to 0} \frac{3x^2}{\sec^2 x} = \lim_{x\to 0} 3x^2\cos^2 x = 0$

4. a. $\int_0^1 x^{-1/3}\,dx = \lim_{a\to 0}\int_a^1 x^{-1/3}\,dx \overset{\lim_{a\to 0}}{=} \frac{3}{2}x^{2/3}\Big|_a^1 = \lim_{a\to 0} \frac{3}{2}(1)^{2/3} - \cancel{\frac{3}{2}(a)^{2/3}}^{\,0} = \boxed{\frac{3}{2}}$ converges

b. $\int_{-\infty}^{1} e^x\,dx = \lim_{a\to-\infty}\int_a^1 e^x\,dx = \lim_{a\to-\infty} e^x\Big|_a^1 = \lim_{a\to-\infty} e^1 - \cancel{e^{-\infty}}^{\,0} = \boxed{e}$ converges

c. $\int_0^1 \frac{e^{1/x}}{x^2}\,dx = \lim_{a\to 0}\int_a^1 \frac{e^{1/x}}{x^2}\,dx = \lim_{a\to 0} -e^{1/x}\Big|_a^1 = \lim_{a\to 0} -e^{1/1} + e^{1/a} = \infty$ $\boxed{\text{diverges}}$

$u = 1/x \quad du = -\frac{1}{x^2}dx$

4d. $\int_1^\infty \frac{e^{-\sqrt{x}}}{\sqrt{x}}\,dx = \lim_{b\to\infty}\int_1^b \frac{e^{-\sqrt{x}}}{\sqrt{x}}\,dx$ $\quad u=-\sqrt{x}$, $du = -\frac{1}{2\sqrt{x}}dx$

$\lim_{b\to\infty} -2e^{-\sqrt{x}}\Big|_1^b = -2e^{-\sqrt{1}} + \cancel{2e^{-\sqrt{b}}}^{\to 0} = \boxed{-\frac{2}{e}}$ converges

e. $\int_2^\infty \frac{1}{\sqrt{x-1}}\,dx = \lim_{b\to\infty}\int_2^b \frac{1}{\sqrt{x-1}}\,dx = \lim_{b\to\infty} 2(x-1)^{1/2}\Big|_2^b = \lim_{b\to\infty} 2(b-1)^{1/2} - 2(2-1)^{1/2}$
$= \infty$ $\boxed{\text{diverges}}$

f. $\int_0^9 \frac{1}{\sqrt[3]{x-1}}\,dx = \int_0^1 \frac{1}{\sqrt[3]{x-1}}\,dx + \int_1^9 \frac{1}{\sqrt[3]{x-1}}\,dx =$

$\lim_{b\to 1}\int_0^b (x-1)^{-1/3}dx + \lim_{a\to 1}\int_a^9 (x-1)^{-1/3}dx = \lim_{b\to 1^-}\frac{3}{2}(x-1)^{2/3}\Big|_0^b + \lim_{a\to 1^+}\frac{3}{2}(x-1)^{2/3}\Big|_a^9$

$= \lim_{b\to 1^-}\cancel{\frac{3}{2}(b-1)^{2/3}}^{0} - \frac{3}{2}(0-1)^{2/3} + \lim_{a\to 1^+}\frac{3}{2}(9-1)^{2/3} - \cancel{\frac{3}{2}(a-1)^{2/3}}^{0} =$

$-3/2 + 4 = \boxed{\frac{10}{3}}$ converges

g. $\int_2^\infty \frac{1}{\sqrt[4]{1+x}}\,dx = \lim_{b\to\infty}\int_2^b (1+x)^{1/4}dx = \lim_{b\to\infty}\frac{4}{5}(1+x)^{5/4}\Big|_2^b = \lim_{b\to\infty}\frac{4}{5}(1+b)^{5/4} - \frac{4}{5}(1+2)^{5/4}$
$= \infty$ $\boxed{\text{diverges}}$

h. $\int_1^\infty \frac{\ln x}{x}\,dx = \lim_{b\to\infty}\int_1^b \frac{\ln x}{x}\,dx = \lim_{b\to\infty}\frac{1}{2}\ln^2 x\Big|_1^b = \lim_{b\to\infty}\frac{1}{2}(\ln b)^2 - \cancel{\frac{1}{2}\ln(1)^2}^{0} = \infty$

$\boxed{\text{diverges}}$

5. $\int_1^\infty \frac{1}{x^p}\,dx$ $\quad$ if $p=1$ $\quad \int_1^\infty \frac{1}{x}\,dx = \ln x\Big|_1^\infty = \infty$ diverges

if $p<1$ $\quad \int_1^\infty \frac{1}{x^p}\,dx = \int_1^\infty x^{-p}dx = \frac{1}{-p+1}x^{-p+1}\Big|_1^\infty \quad -p+1>0 \quad = \infty$ diverges

if $p>1$ $\quad \int_1^\infty \frac{1}{x^p}\,dx = \int_1^\infty x^{-p}dx = \frac{1}{-p+1}x^{-p+1}\Big|_1^\infty \quad -p+1<0 =$ $\boxed{\text{converges}}$

$\cancel{\frac{1}{-p+1}x^{-p+1}}^{0} - \left(\frac{1}{-p+1}(1)^{-p+1}\right)$ finite # $= \frac{1}{-p+1}$

Converges for $p>1$

diverges for $p \le 1$

6. a. convert to sines & cosines / trig integrals

b. trig substitution

c. integration by parts – no tables

d. u-sub $u = 3x+1$

e. u-sub $u = 1-x^2$, possibly a 2nd sub $w = 2-u$

6f. by parts

g. multiply by e^x/e^x, change of variables, w/ partial fractions

h. by parts

i. u-sub $u=\sec\theta$, partial fractions

j. complete square, u-sub?

k. by parts

l. Subst. $u=\sqrt{x}$, factor & partial fractions (may want to use $u=-\sqrt{x}$)

m. u-sub $u=\ln x$, then trig sub or second sub. $w=1+u^2$

n. long division

o. basic rule, e^2 is a constant

p. product-to-sum formula.

q. multiply by e^{-x}/e^{-x}

7. $f(0)=0$, $f(1/2)\approx -.65$, $f(1)=0$, $f(3/2)\approx .4$, $f(2)\approx .6$, $f(5/2)\approx .8$, $f(3)\approx .85$
$f(7/2)\approx .9$, $f(4)\approx .95$, $f(9/2)\approx f(5)\approx 1$.

$\approx \frac{1}{4}\left[0+2(-.65)+2(0)+2(.4)+2(.6)+2(.8)+2(.85)+2(.9)+2(.95)+2(1)+1\right]$

≈ 2.675

8. $f(1770)=5000$, $f(1780)\approx 5000$, $f(1790)\approx 10{,}000$, $f(1800)\approx 13{,}000$ $f(1810)\approx 13{,}000$
$\quad .5 \qquad .5 \qquad 1 \qquad 1.3 \qquad 1.3$

$f(1820)\approx 1$, $f(1830)=1.8$, $f(1840)\approx 3$, $f(1850)=4$, $f(1860)\approx 5$, $f(1870)\approx 6.2$, $f(1880)\approx 8.2$
$f(1890)\approx 8.6$ $f(1900)\approx 9$, $f(1910)\approx 7$ $f(1920)\approx 9.6$, $f(1930)\approx 10.4$, $f(1940)\approx 9.8$,
$f(1950)\approx 9.5$, $f(1960)\approx 8.6$, $f(1970)\approx 9.1$, $f(1980)\approx 9.4$, $f(1990)\approx 10.5$. $f(2000)\approx 12$
$f(2010)\approx 13$

$\approx \frac{10}{3}\left[.5+4(.5)+2(1)+4(1.3)+2(1.3)+4(1)+2(1.8)+4(3)+2(4)+4(5)+2(6.2)+4(8.2)\right.$
$+2(8.6)+4(9)+2(7)+4(9.6)+2(10.4)+4(9.8)+2(9.5)+4(8.6)+2(9.1)$
$\left.+4(9.4)+2(10.5)+4(12)+13\right]\approx$

$\approx 1539.6\overline{6}$ in 10,000s or 15,396,667

9. $x=1$ is in the interval $[0,9]$ and $\frac{x}{x-1}$ not defined there

$\lim_{b\to 1}\int_0^b \frac{x}{x-1}\,dx+\lim_{a\to 1}\int_a^9 \frac{x}{x-1}\,dx$

9b. $\int_0^{\pi} \tan x \, dx$ not defined at $\pi/2$ in interval $[0, \pi]$

$\lim_{b \to \pi/2} \int_0^b \tan x \, dx + \lim_{a \to \pi/2} \int_{\pi/2}^{\pi} \tan x \, dx$

c. $\int_0^{\infty} \frac{1}{1+x^3}$ one limit is ∞

$\lim_{b \to \infty} \int_0^b \frac{1}{1+x^3} \, dx$

d. $\int_{-1}^{1} \frac{dx}{x^2-x-2}$ not defined at $x=-1$

$(x-2)(x+1)$

$\lim_{a \to -1} \int_a^1 \frac{dx}{x^2-x-2}$