192 Homework #2 Key

1a. $\int \cos\theta \cos^5(\sin\theta)\,d\theta$ $u = \sin\theta$, $du = \cos\theta$ $\int \cos^5 u\,du$

$\int \cos^4 u \cos u\,du = \int (1-\sin^2 u)^2 \cos u\,du$ $w = \sin u$, $dw = \cos u\,du$

$\int (1-w^2)^2\,dw = \int 1 - 2w^2 + w^4\,dw = w - \frac{2}{3}w^3 + \frac{1}{5}w^5 + C$

$\boxed{\sin(\sin\theta) - \frac{2}{3}\sin^3(\sin\theta) + \frac{1}{5}\sin^5(\sin\theta) + C}$

b. $\int \frac{dx}{\cos x - 1} \cdot \frac{\cos x + 1}{\cos x + 1} = \int \frac{\cos x + 1}{\cos^2 x - 1}\,dx = -\int \frac{\cos x + 1}{\sin^2 x}\,dx =$

$-\int \csc x \cot x + \csc^2 x\,dx = \boxed{\csc x - \cot x + C}$

c. $\int \frac{\cos x + \sin 2x}{\sin x}\,dx = \int \frac{\cos x + 2\sin x \cos x}{\sin x}\,dx = \int \cot x + 2\cos x\,dx$

$= \boxed{\ln|\sin x| + 2\sin x + C}$

d. $\int_0^{\pi/4} \sqrt{1-\cos 4\theta}\,d\theta =$ $2\sin^2 2\theta = (1-\cos 4\theta)$

$\int_0^{\pi/4} \sqrt{2}\sin 2\theta\,d\theta = -\frac{\sqrt{2}}{2}\cos 2\theta \Big|_0^{\pi/4} = -\frac{\sqrt{2}}{2}[0-1] = \frac{\sqrt{2}}{2}$

e. $\int \frac{\sin^3\sqrt{x}}{\sqrt{x}}\,dx$ $u = \sqrt{x} = x^{1/2}$, $du = \frac{1}{2\sqrt{x}}\,dx$ $\int 2\sin^3 u\,du = 2\int \sin^2 u \sin u\,du$

$2\int (1-\cos^2 u)\sin u\,du$ $w = \cos u$, $dw = -\sin u\,du$

$-2\int 1 - w^2\,dw = -2(w - \frac{1}{3}w^3) + C$

$= \boxed{-2\cos\sqrt{x} + \frac{2}{3}\cos^3\sqrt{x} + C}$

f. $\int \sec^4 q\,dq = \int \sec^2 q \sec^2 q\,dq = \int (1+\tan^2 q)\sec^2 q\,dq$

$\int 1 + u^2\,du = u + \frac{1}{3}u^3 + C = \boxed{\tan q + \frac{1}{3}\tan^3 q + C}$ $u = \tan q$, $du = \sec^2 q\,dq$

192 Homework #2 Key

9. $\int \frac{1}{\tan x+1}\,dx$ $\quad u = \tan x$, $du = \sec^2 x = (\tan^2 x+1)$ $\quad \int \frac{\sec^2 x\,dx}{(\tan x+1)\sec^2 x} = \int \frac{du}{(u+1)(u^2+1)}$

$$\frac{A}{u+1} + \frac{Bu+C}{u^2+1} = \frac{Au^2+A+Bu^2+Cu+Bu+C}{(u+1)(u^2+1)} = \frac{1}{(u+1)(u^2+1)}$$

$A+B=0 \quad (u^2)$

$C+B=0 \quad (u)$

$A+C=1 \quad (\text{const})$

$A=-B$ or $B=-A$

$C-\cancel{A}=0$

$C+\cancel{A}=1$

$2C=1 \Rightarrow C=\frac{1}{2},\ A=\frac{1}{2},\ B=-\frac{1}{2}$

$$\int \frac{1/2}{u+1} + \frac{-\frac{1}{2}u}{u^2+1} + \frac{1/2}{u^2+1}\,du = \frac{1}{2}\ln|u+1| - \frac{1}{4}\ln|u^2+1| + \frac{1}{2}\arctan(u)+C$$

$$= \boxed{\frac{1}{2}\ln|\tan x+1| - \frac{1}{4}\ln|\sec^2 x| + \frac{1}{2}\arctan(\tan x+1) + C}$$

or $\frac{1}{2}\ln|\sec x|$

2a. $\int \tan^5\varphi \sec^4\varphi\,d\varphi = \int \tan^5\varphi(\tan^2\varphi+1)\sec^2\varphi\,d\varphi$ $\quad u=\tan\varphi$, $du=\sec^2\varphi\,d\varphi$

$\boxed{\int u^5(u^2+1)\,du}$

b. $\int \cot^9\varphi \csc^9\varphi\,d\varphi = \int \cot^8\varphi \csc^8\varphi(\csc\varphi\cot\varphi)\,d\varphi =$ $\quad u=\csc\varphi$ $\quad du=-\csc\varphi\cot\varphi\,d\varphi$

$\int (\csc^2\varphi-1)^4\csc^8\varphi(\csc\varphi\cot\varphi)\,d\varphi$

$\boxed{-\int (u^2-1)^4 u^8\,du}$

c. $\int \sin^4\alpha\cos^8 2\alpha\,d\alpha = \int \left[\frac{1}{2}(1-\cos 2\alpha)\right]^2 \cos^8 2\alpha\,d\alpha = \frac{1}{4}\int (1-2\cos 2\alpha+\cos^2 2\alpha)\cos^8 2\alpha\,d\alpha$

$(\sin^2\alpha)^2$

$= \frac{1}{4}\int \underbrace{\cos^8 2\alpha}_{(\cos^2 2\alpha)^4} - \underbrace{2\cos^9 2\alpha}_{\cos 2\alpha(\cos^2 2\alpha)^4} + \underbrace{\cos^{10} 2\alpha}_{(\cos^2 2\alpha)^5}\,d\alpha =$

$-\frac{1}{2}\int \cos 2\alpha(1-\sin^2 2\alpha)^4\,d\alpha + \frac{1}{4}\int \left[\frac{1}{2}(1+\cos 4\alpha)\right]^4 + \left[\frac{1}{2}(1+\cos 4\alpha)\right]^5 d\alpha$

$u=\sin 2\alpha \quad du = 2\cos 2\alpha\,d\alpha$

$-\frac{1}{4}\int (1-u^2)^4\,du$

$\frac{1}{4}\int \frac{1}{16}(1+4\cos 4\alpha+6\cos^2 4\alpha+4\cos^3 4\alpha+\cos^4 4\alpha)\,d\alpha$

$+\frac{1}{4}\int \frac{1}{32}(1+5\cos 4\alpha+10\cos^2 4\alpha+10\cos^3 4\alpha+5\cos^4 4\alpha+\cos^5 4\alpha)\,d\alpha$

$\frac{1}{4}\int \frac{3}{32} + \frac{13}{32}\cos 4\alpha + \frac{9}{16}\cos^3 4\alpha + \frac{5}{32}\cos^5 4\alpha\,d\alpha +$

$\frac{1}{4}\int \frac{11}{16}\cos^2 4\alpha + \frac{7}{32}\cos^4 4\alpha\,d\alpha$

192 Homework #2 Key

2c. cont'd

$$\frac{1}{4}\int \frac{11}{16}\frac{1}{2}(1+\cos 8\alpha)\,d\alpha + \frac{1}{4}\int \frac{7}{32}\frac{1}{2}(1+\cos 8\alpha)^2\,d\alpha$$

$$\frac{11}{128}\int 1+\cos 8\alpha\,d\alpha + \frac{7}{256}\int 1+2\cos 8\alpha+\overbrace{\cos^2 8\alpha}^{\frac{1}{2}(1+\cos 16\alpha)}\,d\alpha$$

$$\int \frac{29}{256}+\frac{9}{64}\cos 8\alpha+\frac{7}{512}+\frac{7}{512}\cos 16\alpha\,d\alpha$$

$w = \sin 4\alpha$
$dw = 4\cos 4\alpha$

$$-\frac{1}{4}\int (1-u^2)^4\,du + \int \frac{77}{512}+\frac{13}{128}\cos 4\alpha+\frac{9}{64}\cos 4\alpha(1-\sin^2 4\alpha)+\frac{5}{128}\cos 4\alpha(1-\sin^2 4\alpha)^2$$
$$+\frac{9}{64}\cos 8\alpha+\frac{7}{512}\cos 16\alpha\,d\alpha$$

$$\boxed{-\frac{1}{4}\int (1-u^2)^4\,du + \int \frac{9}{256}(1-w^2)+\frac{5}{512}(1-w^2)^2\,dw + \int \frac{77}{512}+\frac{13}{128}\cos 4\alpha+\frac{9}{64}\cos 8\alpha+\frac{7}{512}\cos 16\alpha\,d\alpha}$$

$u = \sin 2\alpha$ $\quad$ $w = \sin 4\alpha$

2d. $\int \cos^{10}\beta\,d\beta = \int \left[\frac{1}{2}(1+\cos 2\beta)\right]^5 d\beta = \frac{1}{32}\int 1+5\cos 2\beta+10\cos^2 2\beta+10\cos^3 2\beta+5\cos^4 2\beta+\cos^5 2\beta\,d\beta$

$$\frac{1}{32}\int 1+5\cos 2\beta+10\cos 2\beta\overbrace{(1-\sin^2 2\beta)}^{\cos^2 2\beta}+\cos 2\beta\overbrace{(1-\sin^2 2\beta)^2}^{\cos^4 2\beta} + \frac{1}{32}\int 10\cos^2 2\beta+5\cos^4 2\beta\,d\beta$$

$u = \sin 2\beta \quad du = 2\cos 2\beta$

$$\frac{1}{32}\int \frac{10}{2}(1+\cos 4\beta)+\frac{5}{2}(1+\cos 4\beta)^2\,d\beta$$

$$\frac{1}{32}\int 1+5\cos 2\beta\,d\beta + \frac{1}{64}\int 10(1-u^2)\,du + \frac{1}{64}\int (1-u^2)^2\,du + \frac{10}{64}\int 1+\cos 4\beta\,d\beta +$$
$$\frac{5}{64}\int 1+2\cos 4\beta+\cos^2 4\beta\,d\beta$$

$$\frac{10}{64}\int (1-u^2)\,du + \frac{1}{64}\int (1-u^2)^2\,du + \int \frac{17}{64}+\frac{5}{32}\cos 2\beta+\frac{5}{16}\cos 4\beta+\frac{5}{128}(1+\cos 8\beta)\,d\beta$$

$$\boxed{\frac{10}{64}\int (1-u^2)\,du + \frac{1}{64}\int (1-u^2)^2\,du + \int \frac{39}{128}+\frac{5}{32}\cos 2\beta+\frac{5}{16}\cos 4\beta+\frac{5}{128}\cos 8\beta\,d\beta}$$

$u = \sin 2\beta$

e. $\int \cos^{17}\theta\sin^6\theta\,d\theta = \int (\cos^2\theta)^8\sin^6\theta\cos\theta\,d\theta = \int (1-\sin^2\theta)^8\sin^6\theta\cos\theta\,d\theta$

$u = \sin\theta \quad du = \cos\theta$

$$\boxed{\int (1-u^2)^8 u^6\,du}$$

192 Homework #2 Key

2f. $\int \sin^6\psi \cos^{12}\psi \, d\psi = \int \underline{\sin^6\psi \cos^6\psi} \cos^6\psi \, d\psi$

$\sin 2\psi = 2\sin\psi\cos\psi$

$\frac{1}{2}\sin 2\psi = \sin\psi\cos\psi$

$\int \left(\frac{1}{2}\sin 2\psi\right)^6 \cdot \left[\frac{1}{2}(1+\cos 2\psi)\right]^3 d\psi =$

$\frac{1}{2^9}\int \sin^6 2\psi\,(1+3\cos 2\psi + 3\cos^2 2\psi + \cos^3 2\psi)\, d\psi$

$\frac{1}{512}\int \sin^6 2\psi + 3\cos 2\psi \sin^6 2\psi + 3\sin^6 2\psi \cos^2 2\psi + \sin^6 2\psi \cos^3 2\psi \, d\psi$

$\frac{1}{512}\int 3\cos 2\psi \sin^6 2\psi + \cos 2\psi (1-\sin^2 2\psi)\sin^6 2\psi \, d\psi + \frac{1}{512}\int \sin^6 2\psi + 3\sin^6 2\psi\cos^2 2\psi \, d\psi$

$u = \sin 2\psi$

$du = 2\cos 2\psi$

$\frac{1}{512}\int \left[\frac{1}{2}(1-\cos 4\psi)\right]^3 + 3\left[\left(\frac{1}{2}\right)(1-\cos 4\psi)\right]^3 \frac{(1+\cos 4\psi)}{2} \, d\psi$

$\frac{1}{1024}\int 3u^6 + (1-u^2)u^6 \, du + \frac{1}{4096}\int 1 - 3\cos 4\psi + 3\cos^2 4\psi + \cos^3 4\psi \, d\psi +$

$\frac{3}{8192}\int (1-\cos 4\psi)^2 (1-\cos^2 4\psi)\, d\psi$

$(1 - 2\cos 4\psi + \cos^2 4\psi)(1-\cos^2 4\psi)$

$(1 - \cos^2 4\psi - 2\cos 4\psi + 2\cos^3 4\psi + \cos^2 4\psi - \cos^4 4\psi)$

$(1 - 2\cancel{\cos} 4\psi + 2\cos^3 4\psi - \cos^4 4\psi)$

$=$

$\frac{1}{1024}\int 3u^6 + (1-u^2)u^6 \, du + \int \frac{5}{8192} + \frac{1}{1024}\cos^3 4\psi \, d\psi + \frac{3}{4096}\int \frac{1}{2}(1+\cos 8\psi)\, d\psi +$

$\frac{3}{8192}\int \left[\frac{1}{2}(1+\cos 8\psi)\right]^2 d\psi =$

$\frac{1}{1024}\int 3u^6 + (1-u^2)u^6 \, du + \int \frac{5}{8192} + \frac{1}{1024}\cos 4\psi (1-\sin^2 4\psi)\, d\psi + \frac{3}{8192}\int 1 + \cos 8\psi \, d\psi +$

$w = \sin 4\psi$

$dw = 4\cos 4\psi$

$+\frac{3}{16384}\int 1 + 2\cos 8\psi + \cos^2 8\psi \, d\psi$

$=$

$\frac{1}{1024}\int 3u^6 + (1-u^2)u^6 \, du + \int \frac{19}{16384} + \frac{9}{8192}\cos 8\psi \, d\psi + \frac{1}{4096}\int (1-w^2)\, dw + \frac{3}{16384}\int \left(\frac{1}{2}\right)(1+\cos 16\psi)\, d\psi$

$$\boxed{= \frac{1}{1024}\int 3u^6 + (1-u^2)^6 \, du + \frac{1}{4096}\int (1-w^2)\, dw + \int \frac{41}{32768} + \frac{9}{8192}\cos 8\psi + \frac{3}{32768}\cos 16\psi \, d\psi}$$

g. $\int \cos^{11}\alpha \sin^3 4\alpha \, d\alpha = \int \cos^{11}\alpha (1-\cos^2\alpha)\sin\alpha \, d\alpha$

$u = \cos\alpha$

$du = -\sin\alpha \, d\alpha$

$$\boxed{-\int u^{11}(1-u^2)\, du}$$

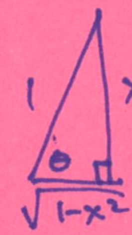
192 Homework #2 key

5a. $\int x^3\sqrt{1-x^2}\,dx$ $\qquad x=\sin\theta$, $dx=\cos\theta\,d\theta$ $\qquad \sqrt{1-x^2}=\sqrt{1-\sin^2\theta}=\sqrt{\cos^2\theta}=\cos\theta$

$$\int \sin^3\theta\cos\theta\cdot\cos\theta\,d\theta = \int \sin\theta(1-\cos^2\theta)\cos^2\theta\,d\theta$$

$u=\cos\theta$, $du=-\sin\theta\,d\theta$ $\qquad \frac{x}{1}=\sin\theta$

$$-\int(1-u^2)u^2\,du = \int u^4-u^2\,du = \frac{1}{5}u^5-\frac{1}{3}u^3+C$$



$$=\frac{1}{5}\cos^5\theta-\frac{1}{3}\cos^3\theta+C$$

$$\boxed{\frac{1}{5}(1-x^2)^{5/2}-\frac{1}{3}(1-x^2)^{3/2}+C}$$

6. $\int\frac{\sqrt{1+x^2}}{x}\,dx$ $\qquad x=\tan\theta$, $dx=\sec^2\theta\,d\theta$ $\qquad \sqrt{1+x^2}=\sqrt{1+\tan^2\theta}=\sqrt{\sec^2\theta}=\sec\theta$

$$\int\frac{\sec\theta\cdot\sec^2\theta\,d\theta}{\tan\theta} = \int\frac{1}{\cos^2\theta}\cdot\frac{\cos\theta}{\sin\theta}\,d\theta = \int\frac{1}{\cos^2\theta}\cdot\frac{1}{\sin\theta}\cdot\frac{\sin\theta}{\sin\theta}\,d\theta =$$

$$\int\frac{1}{\cos^2\theta}\cdot\frac{1}{\sin^2\theta}\cdot\sin\theta\,d\theta = \int\frac{1}{\cos^2\theta}\cdot\frac{1}{(1-\cos^2\theta)}\sin\theta\,d\theta$$

$u=\cos\theta$, $du=-\sin\theta\,d\theta$

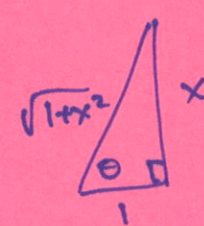
$$+\int\frac{1}{u^2}\cdot\frac{1}{u-1}\cdot\frac{1}{u+1}\,du \qquad \frac{A}{u}+\frac{B}{u^2}+\frac{C}{u-1}+\frac{D}{u+1}=\frac{1}{u^2(u^2-1)}$$

$$\frac{Au(u^2-1)+B(u^2-1)+Cu^2(u+1)+Du^2(u-1)}{u^2(u^2-1)}=\frac{Au^3-Au+Bu^2-B+Cu^3+Cu^2+Du^3-Du^2}{u^2(u^2-1)}$$

$\frac{x}{1}=\tan\theta$

$A+C+D=0$ $\quad(u^3)$
$B+C-D=0$ $\quad(u^2)$
$-A=0$ $\quad(u)$
$-B=1$ $\quad(\text{const})$

$A=0$, $B=-1$ $\Rightarrow$ $C+D=0$, $C-D=1$, $2C=1$, $C=1/2$, $D=-1/2$



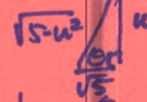
$$\int\frac{-1}{u^2}+\frac{1/2}{u-1}-\frac{\frac{1}{2}}{u+1}\,du = \frac{1}{u}+\frac{1}{2}\ln|u-1|-\frac{1}{2}\ln|u+1|+C$$

$$\frac{1}{\cos\theta}+\frac{1}{2}\ln|\cos\theta-1|-\frac{1}{2}\ln|\cos\theta+1|+C.$$

$=\sec\theta$

$$\boxed{\sqrt{1+x^2}+\frac{1}{2}\ln\left|\frac{1}{\sqrt{1+x^2}}-1\right|-\frac{1}{2}\ln\left|\frac{1}{\sqrt{1+x^2}}+1\right|+C}$$

192 Homework #2 Key

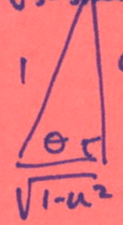


$u=\sqrt{5}\sin\theta \quad du=\sqrt{5}\cos\theta$

$\sqrt{5-u^2}=\sqrt{5-5\sin^2\theta}=\sqrt{5}\cos\theta$

3c. $\displaystyle\int \frac{du}{u\sqrt{5-u^2}} = \int \frac{\sqrt{5}\cos\theta\, d\theta}{\sqrt{5}\sin\theta\,\sqrt{5}\cos\theta} = \int \frac{1}{\sqrt{5}}\csc\theta\, d\theta = \frac{-1}{\sqrt{5}}\ln|\csc\theta+\cot\theta| + C$

$$\boxed{\frac{-1}{\sqrt{5}}\ln\left|\frac{\sqrt{5-u^2}}{u}+\frac{\sqrt{5}}{u}\right|+C}$$



d. $\displaystyle\int x\sqrt{1-x^4}\,dx \qquad u=x^2, \quad du=2x\,dx \qquad \frac{1}{2}\int\sqrt{1-u^2}\,du$

$u=\sin\theta$

$du=\cos\theta\, d\theta$

$\sqrt{1-u^2}=\sqrt{1-\sin^2\theta}$

$=\sqrt{\cos^2\theta}=\cos\theta$

$\displaystyle\frac{1}{2}\int\cos\theta\cdot\cos\theta\, d\theta = \frac{1}{4}\int 1+\cos 2\theta\, d\theta =$

$\frac{1}{4}\theta+\frac{1}{8}\sin 2\theta + C$

$\frac{1}{4}\theta+\frac{1}{8}\cdot 2\sin\theta\cos\theta + C$

$\frac{1}{4}\theta+\frac{1}{4}\sin\theta\cos\theta + C$

$\frac{1}{4}\arcsin u+\frac{1}{4}u\sqrt{1-u^2}+C$

$$\boxed{=\frac{1}{4}\arcsin x^2+\frac{1}{4}x^2\sqrt{1-x^4}+C}$$

e. $\displaystyle\int_0^{\pi/2}\frac{\cos t}{\sqrt{1+\sin^2 t}}\,dt \qquad u=\sin t, \quad du=\cos t\,dt \qquad \int\frac{du}{\sqrt{1+u^2}}$

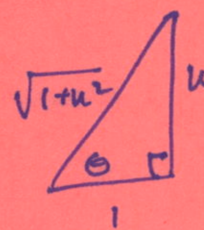
$u=\tan\theta$

$du=\sec^2\theta\, d\theta$

$\sqrt{1+u^2}=\sqrt{1+\tan^2\theta}=\sqrt{\sec^2\theta}$

$=\sec\theta$

$\displaystyle\int\frac{\sec^2\theta}{\sec\theta}\,d\theta = \int\sec\theta\, d\theta = \ln|\sec\theta+\tan\theta|+C$



$\ln|\sqrt{1+u^2}+u|+C$

$$=\boxed{\ln|\sqrt{1+\sin^2 t}+\sin t|+C}$$

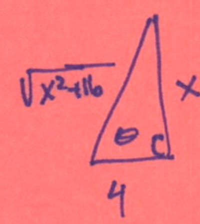
f. $\displaystyle\int\frac{dx}{\sqrt{x^2+16}}$

$x=4\tan\theta$

$dx=4\sec^2\theta\, d\theta$

$\sqrt{x^2+16}=\sqrt{16\tan^2\theta+16}=\sqrt{16\sec^2\theta}=4\sec\theta$

$\frac{x}{4}=\tan\theta$



$\displaystyle\int\frac{\cancel{4}\sec^2\theta}{\cancel{4}\sec\theta}\,d\theta = \int\sec\theta\, d\theta = \ln|\sec\theta+\tan\theta|+C$

$$\boxed{\ln\left|\frac{\sqrt{x^2+16}}{4}+\frac{x}{4}\right|+C}$$

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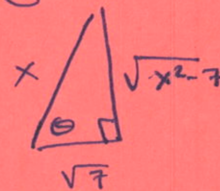
39. $\int \frac{x}{\sqrt{x^2-7}}\,dx$ $\quad x = \sec\theta\cdot\sqrt{7}$ $\quad dx = \sqrt{7}\sec\theta\tan\theta$ $\quad \sqrt{7\sec^2\theta - 7} = \sqrt{7}\tan\theta$ $\quad 7\tan^2\theta$

$$\int \frac{\sqrt{7}\sec\theta\,\cancel{\sqrt{7}}\sec\theta\,\cancel{\tan\theta}\,d\theta}{\cancel{\sqrt{7}}\,\cancel{\tan\theta}} = \sqrt{7}\int \sec^2\theta\,d\theta = \sqrt{7}\tan\theta + C$$

$\frac{x}{\sqrt{7}} = \sec\theta$



$$\cancel{\sqrt{7}}\cdot\frac{\sqrt{x^2-7}}{\cancel{\sqrt{7}}} + C = \boxed{\sqrt{x^2-7} + C}$$

this can also be done w/ regular u-sub.

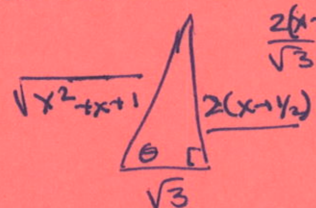
h. $\int \frac{x}{\sqrt{x^2+x+1}}\,dx = \int \frac{x}{\sqrt{(x^2+x+\frac{1}{4})+\frac{3}{4}}}\,dx = \int \frac{x}{\sqrt{(x+\frac{1}{2})^2+\frac{3}{4}}}\,dx$

$(x+\frac{1}{2}) = \frac{\sqrt{3}}{2}\tan\theta \Rightarrow x = \frac{\sqrt{3}}{2}\tan\theta - \frac{1}{2}$

$dx = \frac{\sqrt{3}}{2}\sec^2\theta\,d\theta$ $\quad \sqrt{(x+\frac{1}{2})^2+\frac{3}{4}} = \sqrt{\frac{3}{4}\tan^2\theta+\frac{3}{4}} = \frac{\sqrt{3}}{2}\sec\theta$

$$\int \frac{\left(\frac{\sqrt{3}}{2}\tan\theta - \frac{1}{2}\right)\cancel{\frac{\sqrt{3}}{2}}\sec^{\cancel{2}}\theta\,d\theta}{\cancel{\sqrt{3}/2}\,\cancel{\sec\theta}} = \int \frac{\sqrt{3}}{2}\tan\theta\sec\theta - \frac{1}{2}\sec\theta\,d\theta =$$

$\frac{2(x+\frac{1}{2})}{\sqrt{3}} = \tan\theta$



$$\frac{\sqrt{3}}{2}\sec\theta - \frac{1}{2}\ln|\sec\theta + \tan\theta| + C$$

$$\frac{\cancel{\sqrt{3}}}{2}\cdot\frac{\sqrt{x^2+x+1}}{\cancel{\sqrt{3}}} - \frac{1}{2}\ln\left|\frac{\sqrt{x^2+x+1}}{\sqrt{3}} + \frac{2(x+\frac{1}{2})}{\sqrt{3}}\right| + C$$

$$\boxed{\frac{1}{2}\sqrt{x^2+x+1} - \frac{1}{2}\ln\left|\frac{\sqrt{x^2+x+1}+2(x+\frac{1}{2})}{\sqrt{3}}\right| + C}$$

i. $\int \frac{x^2}{(3+4x-4x^2)^{3/2}}\,dx = \int \frac{x^2}{\left[\sqrt{4-4(x-\frac{1}{2})^2}\right]^3}\,dx$ $\quad -4(x^2-x+\frac{1}{4})+3+1$

$2(x-\frac{1}{2}) = 2\sin\theta$

$$\int \frac{(\sin\theta+\frac{1}{2})^2\cdot\cos\theta\,d\theta}{(2\cos\theta)^3} = \int \frac{(\sin^2\theta+\sin\theta+\frac{1}{4})\cancel{\cos\theta}\,d\theta}{8\cos^{\cancel{3}\,2}\theta}$$

$2x-1 =$

$x-\frac{1}{2} = \sin\theta$

$dx = \cos\theta\,d\theta$

$x = \sin\theta + \frac{1}{2}$

$\sqrt{4-[2(x-\frac{1}{2})]^2} =$

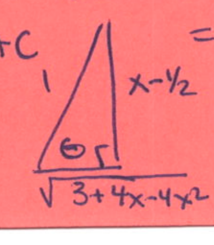
$\sqrt{4-4\sin^2\theta} = \sqrt{4\cos^2\theta}$

$= 2\cos\theta$

$$\frac{1}{8}\int \frac{\sin^2\theta}{\cos^2\theta} + \frac{\sin\theta}{\cos^2\theta} + \frac{1}{4}\sec^2\theta\,d\theta =$$

$$\frac{1}{8}\int \underbrace{\tan^2\theta}_{(\sec^2\theta-1)} + \tan\theta\sec\theta + \frac{1}{4}\sec^2\theta\,d\theta =$$

$$\frac{1}{8}\int \frac{5}{4}\sec^2\theta - 1 + \tan\theta\sec\theta\,d\theta = \frac{1}{8}\left[\frac{5}{4}\tan\theta - \theta + \sec\theta\right] + C$$



$$\boxed{\frac{5}{32}\left[\frac{(x-\frac{1}{2})}{\sqrt{3+4x-4x^2}}\right] - \frac{1}{8}\arcsin(x-\tfrac{1}{2}) + \frac{1}{8}\cdot\frac{1}{\sqrt{3+4x-4x^2}} + C}$$

192 Homework #3 Key

3j: $\int x^2(x^2+1)^{3/2}\,dx$ $\quad x=\tan\theta$, $du=\sec^2\theta\,d\theta$ $\quad \sqrt{x^2+1}=\sqrt{\tan^2\theta+1}=\sqrt{\sec^2\theta}=\sec\theta$

$$\int \tan^2\theta\sec^3\theta\sec^2\theta\,d\theta=\int(\sec^2\theta-1)\sec^5\theta\,d\theta$$

$$=\int\sec^7\theta-\sec^5\theta\,d\theta$$

$\int\sec^7\theta\,d\theta=$

$u=\sec\theta$, $du=\sec\theta\tan\theta\,d\theta$ $\quad dv=\sec^6\theta=\sec^2\theta(\tan^2\theta+1)^2$, $w=\tan\theta$

$v=\int(w^2+1)^2\,dw=\int w^4+2w^2+1\,dw=\frac{1}{5}\tan^5\theta+\frac{2}{3}\tan^3\theta+\tan\theta$

$$\left(\frac{1}{5}\tan^5\theta+\frac{2}{3}\tan^3\theta+\tan\theta\right)\sec\theta-\int\sec\theta\tan\theta\cdot\left(\frac{1}{5}\tan^5\theta+\frac{2}{3}\tan^3\theta+\tan\theta\right)d\theta$$

$$\frac{1}{5}\tan^5\theta\sec\theta+\frac{2}{3}\tan^3\theta\sec\theta+\tan\theta\sec\theta-\int\sec\theta\Big(\frac{1}{5}\underbrace{\tan^6\theta}_{(\sec^2\theta-1)^3}+\frac{2}{3}\underbrace{\tan^4\theta}_{(\sec^2-1)^2}+\tan^2\theta\Big)d\theta$$

$(\sec^2-1)^2=\sec^4\theta-2\sec^2\theta+1$ $\quad (\sec^2\theta-1)^3=\sec^6\theta-3\sec^4\theta+3\sec^2\theta-1$

$$\frac{1}{5}\tan^5\theta\sec\theta+\frac{2}{3}\tan^3\theta\sec\theta+\tan\theta\sec\theta-\frac{1}{5}\int\sec^7\theta-3\sec^5\theta+3\sec^3\theta-\sec\theta\,d\theta+-\frac{2}{3}\int\sec^5\theta-2\sec^3\theta+\sec\theta\,d\theta+-\int\sec^3\theta-\sec\theta\,d\theta$$

$$=\frac{1}{5}\tan^5\theta\sec\theta+\frac{2}{3}\tan^3\theta\sec\theta+\tan\theta\sec\theta-\frac{1}{5}\int\sec^7\theta\,d\theta-\frac{1}{15}\int\sec^5\theta\,d\theta-\frac{4}{15}\int\sec^3\theta\,d\theta+\frac{2}{15}\int\sec\theta\,d\theta=\int\sec^7\theta\,d\theta$$

$+\frac{1}{5}\int\sec^7\theta\,d\theta$ $\qquad +\frac{1}{5}\int\sec^7\theta\,d\theta$

$\Rightarrow \frac{5}{6}$

$$\frac{1}{5}\tan^5\theta\sec\theta+\frac{2}{3}\tan^3\theta\sec\theta+\tan\theta\sec\theta-\frac{1}{15}\int\sec^5\theta\,d\theta-\frac{4}{15}\int\sec^3\theta\,d\theta+\frac{2}{15}\int\sec\theta\,d\theta=\frac{6}{5}\int\sec^7\theta\,d\theta$$

$$\int\sec^7\theta\,d\theta=\frac{1}{6}\tan^5\theta\sec\theta+\frac{5}{9}\tan^3\theta\sec\theta+\frac{5}{6}\sec\theta\tan\theta-\frac{1}{18}\int\sec^5\theta\,d\theta-\underbrace{\frac{4}{18}}_{\frac{2}{9}}\int\sec^3\theta\,d\theta+\underbrace{\frac{2}{18}}_{=\frac{1}{9}}\ln|\sec\theta+\tan\theta|+C$$

$\int\sec^5\theta\,d\theta=$ $\quad u=\sec\theta$, $du=\sec\theta\tan\theta\,d\theta$ $\quad dv=\sec^4\theta=(\tan^2\theta+1)\sec^2\theta$

$w=\tan\theta$ $dw=\sec^2\theta$ $\quad v=\frac{1}{3}\tan^3\theta+\tan\theta$

$$\frac{1}{3}\tan^3\theta\sec\theta+\sec\theta\tan\theta-\int\frac{1}{3}\underbrace{\tan^4\theta}_{(\sec^2\theta-1)^2}\sec\theta+\sec\theta\underbrace{\tan^2\theta}_{(\sec^2\theta-1)}\,d\theta$$

$(\sec^2\theta-1)^2=\sec^4\theta-2\sec^2\theta+1$

$$=\frac{1}{3}\tan^3\theta\sec\theta+\sec\theta\tan\theta-\frac{1}{3}\int\sec^5\theta-2\sec^3\theta+\sec\theta\,d\theta-\int\sec^3\theta-\sec\theta\,d\theta=\int\sec^5\theta\,d\theta$$

$+\frac{1}{3}\int\sec^5\theta$ $\qquad \frac{1}{3}\int\sec^5\theta\,d\theta$

$\Rightarrow \frac{3}{4}$

$$\frac{1}{3}\tan^3\theta\sec\theta+\sec\theta\tan\theta+-\frac{1}{3}\int\sec^3\theta\,d\theta+\frac{2}{3}\int\sec\theta\,d\theta=\frac{4}{3}\int\sec^5\theta\,d\theta$$

$$\int\sec^5\theta\,d\theta=\frac{1}{4}\tan^3\theta\sec\theta+\frac{3}{4}\sec\theta\tan\theta-\frac{1}{4}\int\sec^3\theta\,d\theta+\frac{1}{2}\ln|\sec\theta+\tan\theta|+C$$

$\int\sec^3\theta\,d\theta=$ $\quad u=\sec\theta$, $du=\sec\theta\tan\theta\,d\theta$ $\quad dv=\sec^2\theta\,d\theta$, $v=\tan\theta$

$$\sec\theta\tan\theta-\int\sec\theta\underbrace{\tan^2\theta}_{(\sec^2\theta-1)}\,d\theta$$

$$\sec\theta\tan\theta-\int\sec^3\theta\,d\theta+\int\sec\theta\,d\theta=\int\sec^3\theta\,d\theta$$

$+\int\sec^3\theta\,d\theta$ $\qquad \int\sec^3\theta\,d\theta$

$$\int\sec^3\theta\,d\theta=\frac{1}{2}\sec\theta\tan\theta+\frac{1}{2}\ln|\sec\theta+\tan\theta|+C$$

2j cont'd

$$\int \sec^7\theta - \sec^5\theta \, d\theta =$$

$$\frac{1}{6}\tan^5\theta\sec\theta + \frac{5}{9}\sec\theta\tan^3\theta + \frac{5}{6}\sec\theta\tan\theta - \frac{1}{18}\int\sec^5\theta\,d\theta - \frac{2}{9}\int\sec^3\theta\,d\theta + \frac{1}{9}\ln|\sec\theta+\tan\theta|$$

$$-\int\sec^5\theta\,d\theta$$

$$= \frac{1}{6}\tan^5\theta\sec\theta + \frac{5}{9}\sec\theta\tan^3\theta + \frac{5}{6}\sec\theta\tan\theta - \frac{19}{18}\int\sec^5\theta\,d\theta - \frac{2}{9}\int\sec^3\theta\,d\theta + \frac{1}{9}\ln|\sec\theta+\tan\theta|$$

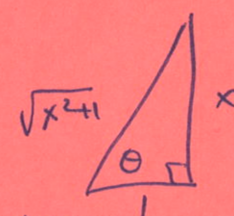
$$= \frac{1}{6}\tan^5\theta\sec\theta + \frac{5}{9}\sec\theta\tan^3\theta + \frac{5}{6}\sec\theta\tan\theta - \frac{19}{18}\left[\frac{1}{4}\tan^3\theta\sec\theta + \frac{3}{4}\sec\theta\tan\theta - \frac{1}{4}\int\sec^3\theta\,d\theta + \frac{1}{2}\ln|\sec\theta+\tan\theta|\right] - \frac{2}{9}\int\sec^3\theta\,d\theta + \frac{1}{9}\ln|\sec\theta+\tan\theta|$$

$$= \frac{1}{6}\tan^5\theta\sec\theta + \frac{7}{24}\tan^3\theta\sec\theta + \frac{1}{24}\sec\theta\tan\theta + \frac{1}{24}\int\sec^3\theta\,d\theta - \frac{23}{36}\ln|\sec\theta+\tan\theta|$$

$$= \frac{1}{6}\tan^5\theta\sec\theta + \frac{7}{24}\tan^3\theta\sec\theta + \frac{1}{24}\sec\theta\tan\theta + \frac{1}{24}\left[\frac{1}{2}\sec\theta\tan\theta + \frac{1}{2}\ln|\sec\theta+\tan\theta|\right] - \frac{23}{36}\ln|\sec\theta+\tan\theta|$$

$$= \frac{1}{6}\tan^5\theta\sec\theta + \frac{7}{24}\tan^3\theta\sec\theta + \frac{1}{16}\sec\theta\tan\theta - \frac{89}{144}\ln|\sec\theta+\tan\theta| + C$$

$$\boxed{\frac{1}{6}x^5\sqrt{x^2+1} + \frac{7}{24}x^3\sqrt{x^2+1} + \frac{1}{16}x\sqrt{x^2+1} - \frac{89}{144}\ln\left|\sqrt{x^2+1}+x\right| + C}$$



4a. $\displaystyle\int \frac{1+6x}{(4x-3)(2x+5)}dx \qquad \frac{A}{4x-3} + \frac{B}{2x+5} = \frac{2Ax+5A+4Bx-3B}{(4x-3)(2x+5)} = \frac{1+6x}{\sim}$

$2A+4B=6$ $\quad A = 11/13$

$5A-3B=1$ $\quad B = 14/13$

$$\frac{11}{13}\int\frac{1}{4x-3}dx + \frac{14}{13}\int\frac{1}{2x+5}dx =$$

$$\frac{11}{13}\cdot\frac{1}{4}\ln|4x-3| + \frac{14}{13}\cdot\frac{1}{2}\ln|2x+5| + C = \boxed{\frac{11}{52}\ln|4x-3| + \frac{7}{13}\ln|2x+5| + C}$$

6. $\displaystyle\int\frac{1}{(x^2-9)^2}dx = \frac{A}{x-3} + \frac{B}{(x-3)^2} + \frac{C}{x+3} + \frac{D}{(x+3)^2}$

$$A(x-3)(x+3)^2 + B(x+3)^2 + C(x-3)^2(x+3) + D(x-3)^2 = 1$$

$x=3$ $\quad B(6)^2 = 1 \Rightarrow B = 1/36$

$x=-3$ $\quad D(-6)^2 = 1 \Rightarrow D = 1/36$

$x=0$ $\quad A(-3)(3)^2 + B(9) + C(-3)^2(3) + D(-3)^2 = 1$

$\quad -27A + \frac{1}{4} + 27C + \frac{1}{4} = 1$

$\quad -27A + 27C = 1/2$

$x=1$ $\quad A(-2)(4)^2 + \frac{4^2}{36} + C(-2)^2(4) + \frac{(-2)^2}{36} = 1$

$\quad -32A + \frac{4}{9} + 16C + \frac{1}{9} = 1$

$\quad -32A + 16C = 4/9$

$\quad A = -1/108, \; C = 1/108$

$$\int\frac{-1/108}{x-3} + \frac{1/36}{(x-3)^2} + \frac{1/108}{x+3} + \frac{1/36}{(x+3)^2}dx =$$

$$\boxed{-\frac{1}{108}\ln|x-3| - \frac{1}{36}\cdot\frac{1}{x-3} + \frac{1}{108}\ln|x+3| - \frac{1}{36}\cdot\frac{1}{x+3} + C}$$

192 Homework #2 Key

4c. $\int \frac{t^6+1}{t^6+t^3}\,dt$

$$t^6+t^3 \overline{)\,t^6+1\,}\quad 1$$
$$-\,(t^6+t^3)$$
$$-t^3+1$$

$$\int 1 + \frac{-t^3+1}{t^3(t^3+1)}\,dt \qquad (t+1)(t^2-t+1)$$

$$\frac{A}{t}+\frac{B}{t^2}+\frac{C}{t^3}+\frac{D}{t+1}+\frac{Et+F}{t^2-t+1}$$

$$At^2(t^3+1)+Bt(t^3+1)+C(t^3+1)+Dt^3(t^2-t+1)+(Et+F)t^3(t+1)$$

$$At^5+At^2+Bt^4+Bt+Ct^3+C+Dt^5-Dt^4+Dt^3+Et^5+Et^4+Ft^4+Ft^3 = -t^3+1$$

$A+D+E=0 \quad (t^5)$
$B-D+E+F=0 \quad (t^4)$
$D+F=-1 \quad (t^3)$
$A+C=0 \quad (t^2)$
$B=0 \quad (t)$
$C=1 \quad$ (const)

$C=1$
$B=0$
$A=1$

$-1+D+E=0 \Rightarrow D+E=1$
$-D+E+F=0$
$D+F=-1$

$D=0$
$E=1$
$F=-1$

$$\int -\frac{1}{t}+\frac{1}{t^3}+\frac{t-1}{t^2-t+1}\,dt$$

$u=t^2-t+1$
$du=2t-1$
$\frac{1}{2}du=t-\frac{1}{2}$

$\left(t^2-t+\frac{1}{4}\right)+\frac{3}{4}$
$\left(t-\frac{1}{2}\right)^2+\frac{3}{4}$

$$\int -\frac{1}{t}+\frac{1}{t^3}+\frac{t-\frac{1}{2}}{t^2-t+1}-\frac{\frac{1}{2}}{\left(t-\frac{1}{2}\right)^2+\frac{3}{4}}\,dt$$

$$-\ln t-\frac{1}{2}t^{-2}+\ln|t^2-t+1|-\frac{1}{2}\cdot\frac{2}{\sqrt{3}}\arctan\left(\frac{t-\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right)+C$$

$$\boxed{-\ln t-\frac{1}{2t^2}+\ln|t^2-t+1|-\frac{1}{\sqrt{3}}\arctan\left(\frac{2t-1}{\sqrt{3}}\right)+C}$$

d. $\int \frac{x^5+x-1}{x^3+1}\,dx$

$$x^3+1\overline{)\,x^5+x-1\,}\quad x^2$$
$$-\,(x^5+x^2)$$
$$-x^2+x-1$$

$$\int x^2+\frac{-x^2+x-1}{(x+1)(x^2-x+1)}\,dx$$

$$\frac{A}{x+1}+\frac{Bx+C}{x^2-x+1} = Ax^2-Ax+A+Bx^2+Bx+Cx+C = -x^2+x+1$$

$A+B=-1$
$-A+B+C=1$
$A+C=1$

$A=-\frac{1}{3}$
$B=-\frac{2}{3}$
$C=\frac{4}{3}$

$\frac{4}{3}-\frac{1}{3}=1$

$u=x^2-x+1$
$du=2x-1$
$\frac{1}{3}du=\frac{2}{3}x-\frac{1}{3}$

$\left(x^2-x+\frac{1}{4}\right)+\frac{3}{4}$
$\left(x-\frac{1}{2}\right)^2+\frac{3}{4}$

$$\int x^2-\frac{\frac{1}{3}}{x+1}+\frac{-\frac{2}{3}x+\frac{1}{3}}{x^2-x+1}+\frac{1}{x^2-x+1}\,dx = \int x^2-\frac{\frac{1}{3}}{x+1}+\frac{-\frac{2}{3}x+\frac{1}{3}}{x^2-x+1}+\frac{1}{\left(x-\frac{1}{2}\right)^2+\frac{3}{4}}\,dx$$

$$\boxed{\frac{1}{3}x^3-\frac{1}{3}\ln|x+1|-\frac{1}{3}\ln|x^2-x+1|+\frac{2}{\sqrt{3}}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)+C}$$

192 Homework #2 Key

4e. $\displaystyle\int \frac{e^{2x}}{e^{2x}+3e^x+2}\,dx$ ($e^{2x} = e^x \cdot e^x$) let $u = e^x$, $du = e^x\,dx$ $\displaystyle\int \frac{u\,du}{u^2+3u+2} = \int \frac{u}{(u+2)(u+1)}\,du$

$$\frac{A}{u+2} + \frac{B}{u+1} = \frac{Au+A+Bu+2B}{(u+2)(u+1)} = u \qquad \begin{aligned} A+B&=1 \\ A+2B&=0 \end{aligned} \qquad \begin{aligned} A&=2 \\ B&=-1 \end{aligned}$$

$$\int \frac{2}{u+2} - \frac{1}{u+1}\,du = 2\ln|u+2| - \ln|u+1| + C$$

$$= \boxed{2\ln|e^x+2| - \ln|e^x+1| + C}$$

f. $\displaystyle\int \frac{x^4+1}{x^3(x^2+4)}\,dx = \int \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{Dx+E}{x^2+4}\,dx$

$$Ax^2(x^2+4) + Bx(x^2+4) + C(x^2+4) + (Dx+E)x^3 = x^4+1$$

$$Ax^4 + 4Ax^2 + Bx^3 + 4Bx + Cx^2 + 4C + Dx^4 + Ex^3 = x^4+1$$

$A + D = 1 \Rightarrow D = 17/16$

$B + E = 0 \Rightarrow E = 0$

$4A + C = 0 \Rightarrow 4A = -1/4 \Rightarrow A = -1/16$

$4B = 0 \Rightarrow B = 0$

$4C = 1 \Rightarrow C = 1/4$

$$\int \frac{-1/16}{x} + \frac{1/4}{x^3} + \frac{17/16}{x^2+4}\,dx = \boxed{-\frac{1}{16}\ln|x| - \frac{1}{8}\cdot\frac{1}{x^2} + \frac{17}{32}\arctan\left(\frac{x}{2}\right) + C}$$

g. $\displaystyle\int \frac{x^4-2x^3+x^2+2x-1}{x^2-2x+1}\,dx$

$$\begin{array}{r} x^2 \\ x^2-2x+1 \,\big)\, \overline{x^4-2x^3+x^2+2x-1} \\ -\,(x^4-2x^3+x^2) \\ \hline 2x-1 \end{array}$$

$$\int x^2 + \frac{2x-1}{(x-1)^2}\,dx \qquad \frac{A}{x-1} + \frac{B}{(x-1)^2} = Ax - A + B = 2x-1 \qquad \begin{aligned} -A+B&=-1 \\ -2+B&=-1 \end{aligned}$$

$A = 2$, $B = 1$

$$\int x^2 + \frac{2}{x-1} + \frac{1}{(x-1)^2}\,dx =$$

$$\boxed{\frac{1}{3}x^3 + 2\ln|x-1| - \frac{1}{x-1} + C}$$

h. $\displaystyle\int \frac{4x}{x^3+x^2+x+1}\,dx$ $x^3+x^2+x+1 = x^2(x+1) + 1(x+1) = (x^2+1)(x+1)$

$$\frac{Ax+B}{x^2+1} + \frac{C}{x+1} \Rightarrow Ax^2 + Bx + Ax + B + Cx^2 + C = 4x$$

$A + C = 0$ $B + C = 0$ $A = 2,\ B = 2,\ C = -2$

$A + B = 4$

$$\int \frac{2x}{x^2+1} + \frac{2}{x^2+1} + \frac{-2}{x+1}\,dx = \boxed{\ln|x^2+1| + 2\arctan x - 2\ln|x+1| + C}$$

192 Homework #2 key

4i. $\int \frac{x^3+2x}{x^4+4x^2+3}\,dx$ $\quad \cancel{(x^4+4x^2+4)-1 = (x^2+2)^2-1}$ $\quad (x^2+3)(x^2+1)$

$\int \frac{Ax+B}{x^2+3} + \frac{Cx+D}{x^2+1}\,dx$ $\quad x^3+2x = Ax^3+Bx^2+Ax+B+Cx^3+Dx^2+3Cx+3D$

$A+C=1$ $\quad B=D=0$

$\cancel{B+D=0}$ $\quad A=1/2 \quad C=1/2$

$A+3C=2$

$\cancel{B+3D=0}$

$\int \frac{1/2\,x}{x^2+3} + \frac{1/2\,x}{x^2+1}\,dx$

$\frac{1}{2}\cdot\frac{1}{2}\ln|x^2+3| + \frac{1}{2}\cdot\frac{1}{2}\ln|x^2+1| + C = \boxed{\frac{1}{4}\ln|x^2+3| + \frac{1}{4}\ln|x^2+1| + C}$

5a. $\int \frac{dx}{(x+3)^2(x-2)(x^2+4)x^3} = \int \frac{A}{x+3} + \frac{B}{(x+3)^2} + \frac{C}{x-2} + \frac{Dx+E}{x^2+4} + \frac{F}{x} + \frac{G}{x^2} + \frac{H}{x^3}\,dx$

$$\boxed{A\ln|x+3| - \frac{B}{(x+3)} + C\ln|x-2| + \frac{D}{2}\ln|x^2+4| + \frac{E}{2}\arctan\left(\frac{x}{2}\right) + F\ln|x| - \frac{G}{x} - \frac{H}{2x^2} + C}$$

b. $\int \frac{dx}{(x^2-4)(x^2+7)^2(x-1)^3(x+1)} = \int \frac{A}{x-2} + \frac{B}{x+2} + \frac{Cx+D}{x^2+7} + \frac{Ex+F}{(x^2+7)^2} + \frac{G}{x-1} + \frac{H}{(x-1)^2}$

$+ \frac{I}{(x-1)^3} + \frac{J}{x+1}\,dx =$ $\boxed{A\ln|x-2| + B\ln|x+2| + \frac{C}{2}\ln|x^2+7| + \frac{D}{\sqrt{7}}\arctan\left(\frac{x}{\sqrt{7}}\right)}$

$$\boxed{+\frac{1}{98}\left(\frac{7(Fx-7E)}{x^2+7} + \sqrt{7}F\arctan\left(\frac{x}{\sqrt{7}}\right)\right) + G\ln|x-1| - \frac{H}{x-1} - \frac{I}{2(x-1)^2} + J\ln|x+1|}$$

c. $\int \frac{dx}{(x^2+2x+2)(x^2+4x+3)^2(x-1)(x+2)^5} = \int \frac{Ax+B}{(x+1)^2+1} + \frac{C}{(x+3)} + \frac{D}{(x+3)^2} + \frac{E}{(x+1)} + \frac{F}{(x+1)^2} + \frac{G}{x-1}$

$(x^2+2x+1)+1$ $\quad (x+3)^2(x+1)^2$

$(x+1)^2+1$

$+ \frac{H}{x+2} + \frac{I}{(x+2)^2} + \frac{J}{(x+2)^3} + \frac{K}{(x+2)^4} + \frac{L}{(x+2)^5}\,dx =$

$$= \boxed{\begin{aligned}&(B-A)\arctan(x+1) + \tfrac{1}{2}A\ln(x^2+2x+2) + C\ln|x+3| - \frac{D}{x+3} + E\ln|x+1| - \frac{F}{x+1} + \\ &G\ln|x-1| + H\ln|x+2| - \frac{I}{x+2} - \frac{J}{2(x+2)^2} - \frac{K}{3(x+2)^3} - \frac{L}{4(x+2)^4} + C\end{aligned}}$$

6a. $\int \frac{1}{3\sin x - 4\cos x}\,dx$ $\quad \cos x = \frac{1-t^2}{1+t^2}$, $\sin x = \frac{2t}{1+t^2}$ $\quad dx = \frac{2\,dt}{t^2+1}$

$\int \frac{1}{\frac{3(1-t^2)}{\cancel{1+t^2}} - \frac{4(2t)}{\cancel{1+t^2}}}\,\frac{2\,dt}{\cancel{t^2+1}} = \int \frac{2}{3(1-t^2)-8t}\,dt = \int \frac{2}{3-8t-3t^2}\,dt =$

$(3+t)(1-3t)$

6a cont'd.

$\tan x = \frac{\sin x}{\cos x} = \frac{2t}{1-t^2}$

$t = \tan\left(\frac{x}{2}\right)$

$\int \frac{A}{t+3} + \frac{B}{1-3t}\, dt$

$A - 3At + Bt + 3B = 2$

$-3A + B = 0$ $\quad A = 1/5$

$A + 3B = 2$ $\quad B = 3/5$

$\int \frac{1/5}{t+3} + \frac{3/5}{1-3t}\, dt = \frac{1}{5}\ln|t+3| + \frac{3}{5}\ln|1-3t| + C = \ln\left|\sqrt[5]{t+3}\cdot\sqrt[5]{(1-3t)^3}\right| + C$

$\frac{1}{5}\ln\left|\tan\left(\frac{x}{2}\right)+3\right| + \frac{3}{5}\ln\left|1-3\tan\frac{x}{2}\right| + C$

b. $\int \frac{1}{1+\sin x - \cos x}\, dx = \int \frac{1}{1+\frac{2t}{1+t^2} - \frac{1-t^2}{1+t^2}} \cdot \frac{2}{1+t^2}\, dt = \int \frac{2}{1+t^2+2t-1+t^2}\, dt$

$= \int \frac{2}{2t^2+2t}\, dt = \int \frac{1}{t^2+t}\, dt = \int \frac{A}{t} + \frac{B}{t+1}\, dt = \int \frac{1}{t} - \frac{1}{t+1}\, dt$

$At + A + Bt = 1$

$A + B = 0$

$A = 1 \Rightarrow B = -1$

$\boxed{\ln|t| - \ln|t+1| + C}$

7a. $\int \frac{\sqrt{x+1}}{x}\, dx$ $\quad u = \sqrt{x+1}$

$u^2 = x+1 \Rightarrow u^2 - 1 = x$

$2u\,du = dx$

$\boxed{\int \frac{u\cdot 2u\,du}{u^2-1}}$

b. $\int \frac{x^3}{\sqrt[3]{x^2+1}}\, dx$ $\quad u = \sqrt[3]{x^2+1}$

$u^3 = x^2+1 \Rightarrow u^3 - 1 = x^2$

$\int \frac{x^2\cdot x\,dx}{\sqrt[3]{x^2+1}}$ $\quad 3u^2\,du = 2x\,dx \Rightarrow \frac{3}{2}u^2\,du = x\,dx$

$\boxed{\int \frac{(u^3-1)\frac{3}{2}u^2\,du}{u}}$

c. $\int \frac{1}{\sqrt{x}+\sqrt[3]{x}}\, dx$ $\quad u = \sqrt[6]{x}$ $\quad \sqrt{x} = u^3$

$u^6 = x$ $\quad \sqrt[3]{x} = u^2$

$6u^5\,du = dx$

$\boxed{\int \frac{6u^5\,du}{u^3+u^2}}$

d. $\int \frac{dx}{1+e^x} \cdot \frac{e^{-x}}{e^{-x}} = \int \frac{e^{-x}\,dx}{e^{-x}+1}$ $\quad u = e^{-x}+1$

$du = -e^{-x}\,dx$

$\boxed{\int \frac{-du}{u}}$

e. $\int \frac{1}{1+\sqrt[3]{x}}\, dx$ $\quad u = \sqrt[3]{x}$

$u^3 = x$

$3u^2\,du = dx$

$\boxed{\int \frac{3u^2\,du}{1+u}\, du}$

f. $\int \frac{\sqrt{x}}{x^2+x}\, dx$ $\quad u = \sqrt{x}$ $\quad 2u\,du = dx$

$u^2 = x$

$\boxed{\int \frac{u\cdot 2u\,du}{u^4+u^2}}$

7g. $\int \frac{\sqrt{1+\sqrt{x}}}{x}\,dx$

$u=\sqrt{1+\sqrt{x}}$
$u^2=1+\sqrt{x}$
$u^2-1=\sqrt{x}$
$(u^2-1)^2=x$
$u^4-2u^2+1=x$
$(4u^3-4u)\,du=dx$

$\boxed{\int \frac{u\cdot(4u^3-4u)\,du}{u^4-2u^2+1}}$

8a. $\int \operatorname{sech}^3 x \tanh x\,dx = \int \operatorname{sech}^2 h\,(\operatorname{sech} x \tanh x)\,dx$ $u=\operatorname{sech} x$, $-du=\operatorname{sech} x\tanh x\,dx$

$-\int u^2\,du = -\frac{1}{3}u^3+C$

$\boxed{-\frac{1}{3}\operatorname{sech}^3 x + C}$

b. $\int t\cot t\csc t\,dt$ $u=t$, $du=dt$ $dv=\cot t\csc t$, $v=-\csc t$

$-t\csc t+\int t\csc t\,dt$

$\boxed{-t\csc t-\ln|\csc t+\cot t|+C}$

c. $\int x\operatorname{arcsec} x\,dx$ $u=\operatorname{arcsec} x$, $du=\frac{1}{x\sqrt{x^2-1}}\,dx$ $dv=x$, $v=\frac{1}{2}x^2$

$\frac{1}{2}x^2\operatorname{arcsec} x-\frac{1}{2}\int\frac{x^{\cancel{2}}}{\cancel{x}\sqrt{x^2-1}}\,dx$ $w=x^2-1$, $dw=2x\,dx$

$-\frac{1}{2}\int\frac{\frac{1}{2}\,dw}{w^{1/2}}=-\frac{1}{4}\int w^{-1/2}\,dw$

$-\frac{1}{4}\cdot 2w^{1/2}$

$\boxed{\frac{1}{2}x^2\operatorname{arcsec} x-\frac{1}{2}\sqrt{x^2-1}+C}$

d. $\int_0^{\pi/3}\tan^2 x\,dx=\int_0^{\pi/3}\sec^2 x-1\,dx=\tan x-x\Big|_0^{\pi/3}=\boxed{\sqrt{3}-\pi/3}$

e. $\int\cos^4 x\,dx=\int\left[\frac{1}{2}(1+\cos 2x)\right]^2dx=\frac{1}{4}\int 1+2\cos 2x+\cos^2 2x\,dx=$

$\frac{1}{4}\int 1+2\cos 2x+\frac{1}{2}(1+\cos 4x)\,dx=\frac{1}{4}\int\frac{3}{2}+2\cos 2x+\frac{1}{2}\cos 4x\,dx=$

$\boxed{\frac{1}{4}\left[\frac{3}{2}x+\sin 2x+\frac{1}{8}\sin 4x\right]+C}$

f. $\int\frac{\cot^3\theta}{\csc\theta}\,d\theta\cdot\frac{\csc\theta}{\csc\theta}=\int\frac{\cot^3\theta\csc\theta}{1+\cot^2\theta}\,d\theta=\int\frac{\csc\theta\cot\theta(\csc^2\theta-1)}{\csc^2\theta}\,d\theta$

$u=\csc\theta$, $du=-\csc\theta\cot\theta\,d\theta$

$-\int\frac{u^2-1}{u^2}\,du=\int\frac{1-u^2}{u^2}\,du=\int\frac{1}{u^2}-1\,du=-\frac{1}{u}-u+C$

$\boxed{-\sin\theta-\csc\theta+C}$

8g. $\int e^{2x}\sqrt{1+e^{2x}}\,dx$ $\quad u=e^{2x}$, $du=2e^{2x}dx$ $\quad \int \frac{1}{2}\sqrt{1+u}\,du = \frac{1}{2}\int (1+u)^{1/2}du$

$\frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{3}(1+u)^{3/2}+C = \boxed{\frac{1}{3}(1+e^{2x})^{3/2}+C}$

h. $\int \frac{4x^2}{x^3+x^2-x-1}dx = \int \frac{4x^2}{(x-1)(x+1)^2}dx = \int \frac{A}{x-1}+\frac{B}{x+1}+\frac{C}{(x+1)^2}dx$

$x^2(x+1)-1(x+1)$
$(x^2-1)(x+1)$

$A(x+1)^2+B(x-1)(x+1)+C(x-1)=4x^2$

$x=-1$ $\quad -2C=4 \Rightarrow C=-2$

$x=1$ $\quad 4A=4 \Rightarrow A=1$

$x=0$ $\quad A+(-B)-C=0$

$1-B+2=0 \Rightarrow B=3$

$\int \frac{1}{x-1}+\frac{3}{x+1}-\frac{2}{(x+1)^2}dx$

$\boxed{\ln|x-1|+3\ln|x+1|+\frac{2}{x+1}+C}$

i. $\int \frac{\sec^2 x\,dx}{\tan x(\tan x+1)}$ $\quad u=\tan x$, $du=\sec^2 x\,dx$ $\quad \int \frac{du}{u(u+1)} = \int \frac{A}{u}+\frac{B}{u+1}du$

$Au+A+Bu=1$
$A+B=0$
$A=1 \Rightarrow B=-1$

$\int \frac{1}{u}-\frac{1}{u+1}du = \ln u - \ln|u+1|+C$

$= \boxed{\ln|\tan x| - \ln|\tan x+1|+C}$

j. $\int x\sqrt{4-x}\,dx$ $\quad u=\sqrt{4-x}$, $u^2=4-x$, $x=4-u^2$, $dx=-2u\,du$

$\int (4-u^2)(-2u\,du)\cdot u$

$=\int (u^2-4)2u^2\,du = \int 2u^4-8u^2\,du$

$\frac{2}{5}u^5-\frac{8}{3}u^3+C = \boxed{\frac{2}{5}(4-x)^{5/2}-\frac{8}{3}(4-x)^{3/2}+C}$

k. $\int 4\arccos x\,dx$ $\quad u=\arccos x$, $du=\frac{-1}{\sqrt{1-x^2}}dx$ $\quad dv=dx$, $v=x$

$4x\arccos x+\int \frac{x}{\sqrt{1-x^2}}dx$ $\quad w=1-x^2$, $dw=-2x\,dx$ $\quad -\frac{1}{2}\int w^{-1/2}dw$

$\boxed{4x\arccos x-\sqrt{1-x^2}+C}$

l. $\int x^2(x-2)^{3/2}dx$ $\quad u=\sqrt{x-2}$, $u^2=x-2$, $u^2+2=x$, $2u\,du=dx$, $u^4+4u^2+4=x^2$

$\int (u^4+4u^2+4)u^3\cdot 2u\,du = \int 2u^8+8u^6+8u^4\,du$

$\frac{2}{9}u^9+\frac{8}{7}u^7+\frac{8}{5}u^5+C$

$\boxed{\frac{2}{9}(x-2)^{9/2}+\frac{8}{7}(x-2)^{7/2}+\frac{8}{5}(x-2)^{5/2}+C}$

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8m. $\int \sin^5 x \cos^2 x \, dx = \int \sin x (1-\cos^2 x)^2 \cos^2 x \, dx$ $\quad u = \cos x$, $du = -\sin x \, dx$

$-\int (1-u^2)^2 u^2 \, du = -\int (1-2u^2+u^4) u^2 \, du =$

$-\int u^2 - 2u^4 + u^6 \, du = -\frac{1}{3}u^3 + \frac{2}{5}u^5 - \frac{1}{7}u^7 + C$

$$\boxed{-\frac{1}{3}\cos^3 x + \frac{2}{5}\cos^5 x - \frac{1}{7}\cos^7 x + C}$$

n. $\int \sin x \tan^2 x \, dx = \int \frac{\sin^3 x}{\cos^2 x} dx = \int \frac{\sin x (1-\cos^2 x)}{\cos^2 x} dx$ $\quad u = \cos x$, $du = -\sin x \, dx$

$-\int \frac{1-u^2}{u^2} du = \int -\frac{1}{u^2} + 1 \, du = \frac{1}{u} + u + C = \boxed{\sec\theta + \cos\theta + C}$

o. $\int \frac{\sqrt{x^2+16}}{x} dx$ $\quad x = 4\tan\theta$, $dx = 4\sec^2\theta \, d\theta$ $\quad \sqrt{x^2+16} = \sqrt{16\tan^2\theta + 16} = \sqrt{16\sec^2\theta} = 4\sec\theta$

$\int \frac{4\sec\theta \cdot \cancel{4}\sec^2\theta}{\cancel{4}\tan\theta} d\theta = 4\int \frac{1}{\underset{2}{\cos^3\theta}} \cdot \frac{\cos\theta}{\sin\theta} d\theta = 4\int \frac{1}{\cos^2\theta} \cdot \frac{1}{\sin\theta} \cdot \frac{\sin\theta}{\sin\theta} d\theta =$

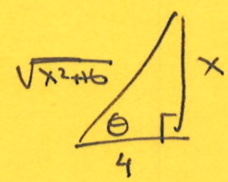
$\int \frac{\sin\theta}{\cos^2\theta (1-\cos^2\theta)} d\theta$ $\quad u = \cos\theta$, $du = -\sin\theta \, d\theta$ $\quad +\int \frac{+1}{u^2(-1+u^2)} du = \int \frac{A}{u} + \frac{B}{u^2} + \frac{C}{u+1} + \frac{D}{u-1} du$

$Au(u^2-1) + B(u^2-1) + Cu^2(u-1) + D(u+1)u^2 = 1$ $\qquad \frac{x}{4} = \tan\theta$

$u=0 \quad -B = 1 \Rightarrow B = -1$

$u=1 \quad D(2) = 1 \Rightarrow D = \frac{1}{2}$

$u=-1 \quad C(-2) = 1 \Rightarrow C = -\frac{1}{2}$



$u=2 \quad A(2)(3) - 1(3) - \frac{1}{2}(4)(1) + \frac{1}{2}(3)(4) = 1$

$6A - 3 - 2 + 6 = 1 \Rightarrow 6A + 1 = 1$

$6A = 0 \Rightarrow A = 0$

$\int \frac{-1}{u^2} - \frac{\frac{1}{2}}{u+1} + \frac{\frac{1}{2}}{u-1} du = \frac{1}{u} - \frac{1}{2}\ln|u+1| + \frac{1}{2}\ln|u-1| + C$

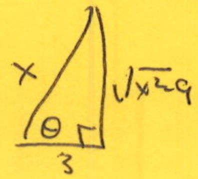
$\sec\theta - \frac{1}{2}\ln|\cos\theta + 1| + \frac{1}{2}\ln|\cos\theta - 1| + C$

$$\boxed{\frac{\sqrt{x^2+16}}{4} - \frac{1}{2}\ln\left|\frac{4}{\sqrt{x^2+16}} + 1\right| + \frac{1}{2}\ln\left|\frac{4}{\sqrt{x^2+16}} - 1\right| + C}$$

p. $\int_3^6 \frac{\sqrt{x^2-9}}{x^2} dx$ $\quad x = 3\sec\theta$, $dx = 3\sec\theta\tan\theta \, d\theta$ $\quad \sqrt{x^2-9} = \sqrt{9\sec^2\theta - 9} = \sqrt{9\tan^2\theta} = 3\tan\theta$

$\int \frac{\cancel{3}\tan\theta \cdot \cancel{3}\cancel{\sec\theta}\tan\theta \, d\theta}{\cancel{9}\sec^{\cancel{2}}\theta} = \int \frac{\tan^2\theta \, d\theta}{\sec\theta} = \int \frac{\sec^2\theta - 1}{\sec\theta} d\theta = \int \sec\theta - \cos\theta \, d\theta$ $\quad \sec\theta = \frac{x}{3}$

$\ln|\sec\theta + \tan\theta| - \sin\theta + C \Rightarrow \ln\left|\frac{x}{3} - \frac{\sqrt{x^2-9}}{3}\right| - \frac{\sqrt{x^2-9}}{x}\Big|_3^6$



$\ln\left|2 - \frac{\sqrt{27}}{3}\right| - \frac{\sqrt{27}}{6} - \cancel{\ln\left|1 - \frac{0}{3}\right|} - \cancel{\frac{0}{3}} = \boxed{\ln|2-\sqrt{3}| - \frac{\sqrt{3}}{2}}$

8q. $\int_0^1 \frac{x^2-x}{x^2+x+1}\,dx$

$$x^2+x+1 \overset{\displaystyle 1}{\overline{\big)\, x^2 - x}}$$
$$\underline{-\;x^2 \mp x \mp 1}$$
$$-2x-1$$

$\int 1 - \left(\frac{2x+1}{x^2+x+1}\right) dx$

$du = 2x+1\,dx$

$u = x^2+x+1$

$x - \ln|x^2+x+1|\Big|_0^1 = 1 - \ln|1+1+1| - 0 + \cancel{\ln(1)}^{0} = \boxed{1-\ln(3)}$

r. $\int \frac{1}{t[1+(\ln t)^2]}\,dt$ $\quad u = \ln t$, $du = \frac{dt}{t}$

$\int \frac{1}{1+u^2}\,du = \arctan u + C$

$= \boxed{\arctan(\ln t) + C}$