

192 Homework #1 Key

①

1. a. $\frac{1}{x^2} \cdot 2x = \frac{2}{x}$ (Second Fundamental Theorem of Calculus).

b. $x^4 \cos(x^8) \cdot 4x^3 = 4x^7 \cos(x^8)$

c. $2xe^{x^2} \int_0^x e^{-t^2} dt + e^{x^2} \cdot e^{-x^2} = 2xe^{x^2} \int_0^x e^{-t^2} dt + 1$

d. $e^{\cos(\sin^2 3t)} \cdot (-\sin(\sin^2 3t)) \cdot 2 \sin 3t \cos 3t \cdot 3$

e. $g(t) = \cos(\ln t) t^{-1}$ $g'(t) = -\sin(\ln t) \cdot \frac{1}{t} \cdot \frac{1}{t} - \frac{1}{t^2} \cos(\ln t)$
 $= -\frac{1}{t^2} (\sin(\ln t) + \cos(\ln t))$

f. $\sec^2\left(\frac{1}{t}\right) \left(-\frac{1}{t^2}\right)$

g. $-\sqrt{x + \sin x}$

h. $\frac{e^x}{x} - \frac{e^{\sqrt{x}}}{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} = \frac{e^x}{x} - \frac{e^{\sqrt{x}}}{2x}$

i. $2^{t-1} \cdot \ln 2$

j. $\frac{1}{2} (x^2+3)^{-1/2} \cdot 2x = \frac{x}{\sqrt{x^2+3}}$

k. $\frac{1}{\sqrt{1-(e^t+t)^2}} \cdot (e^t+1) = \frac{e^t+1}{\sqrt{1-(e^t+t)^2}}$

2a. $\int \frac{x^2-2x}{x^3-3x^2+3x-1}$

$u = x^3 - 3x^2 + 3x - 1$
 $du = (3x^2 - 6x + 3) dx$

$\int \frac{x^2-2x+1}{x^3-3x^2+3x-1} \cdot \frac{-1}{(x-1)^3} dx$

$\frac{1}{3} du = (x^2-2x+1) dx$

$\frac{1}{3} \int \frac{du}{u} - \int \frac{1}{(x-1)^3} dx \quad (x-1)^{-3}$

$\frac{1}{3} \ln u - \frac{1}{2} (x-1)^{-2} + C \Rightarrow \frac{1}{3} \ln |x-1|^3 - \frac{1}{2(x-1)^2} + C$

$= \boxed{\ln|x-1| - \frac{1}{2(x-1)^2} + C}$

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$$2b. \int \frac{1}{x \ln(x^3)} dx = \int \frac{1}{3x \ln x} dx \quad \begin{array}{l} u = \ln x \\ du = \frac{1}{x} dx \end{array}$$

$$\frac{1}{3} \int \frac{1}{u} du = \frac{1}{3} \ln u + C = \boxed{\frac{1}{3} \ln(\ln(x^3)) + C}$$

$$c. \int \frac{1}{(x-1)\sqrt{x^2-2x}} dx = \int \frac{1}{(x-1)\sqrt{(x^2-2x+1)-1}} dx = \int \frac{1}{(x-1)\sqrt{(x-1)^2-1}} dx$$

$$u = x-1 \quad \begin{array}{l} du = dx \\ \int \frac{1}{u\sqrt{u^2-1}} du = \operatorname{arcsec} u + C = \boxed{\operatorname{arcsec}(x-1) + C} \end{array}$$

$$d. \int \frac{\operatorname{sinh}(x)}{1 + \operatorname{sinh}^2 x} dx = \int \frac{\operatorname{sinh} x}{\cosh^2 x} dx = \int \tanh x \operatorname{sech} x dx =$$

$$\boxed{-\operatorname{sech} x + C}$$

$$e. \int \tan x \ln(\cos x) dx \quad \begin{array}{l} u = \ln(\cos x) \\ -du = +\tan x dx \end{array}$$

$$-\int u du = -\frac{1}{2} u^2 + C = \boxed{-\frac{1}{2} \ln^2(\cos x) + C}$$

$$f. \int \frac{1}{3t+1} - \frac{17}{(4t-1)^2} dt$$

$$u = 3t+1 \quad u = 4t-1$$

$$du = 3dt \quad du = 4dt$$

$$\frac{1}{3} du = dt \quad \frac{1}{4} du = dt$$

$$\frac{1}{3} \int \frac{1}{u} du \quad -\frac{17}{4} \int u^{-2} du$$

$$= \boxed{\frac{1}{3} \ln|3t+1| + \frac{17}{4} \frac{1}{4t-1} + C}$$

$$g. \int \frac{\sec x \tan x}{\sec x - 1} dx \quad \begin{array}{l} u = \sec x - 1 \\ du = \sec x \tan x \end{array}$$

$$\int \frac{1}{u} du = \ln u + C = \boxed{\ln|\sec x - 1| + C}$$

$$h. \int_{-2}^2 \frac{dx}{x^2+4x+13} = \int_{-2}^2 \frac{dx}{x^2+4x+4+9} = \int_{-2}^2 \frac{dx}{(x+2)^2+9} = \frac{1}{3} \operatorname{arctan}\left(\frac{x+2}{3}\right) \Big|_{-2}^2 =$$

$$\frac{1}{3} \operatorname{arctan}\left(\frac{4}{3}\right) - \frac{1}{3} \operatorname{arctan}(0) = \boxed{\frac{1}{3} \operatorname{arctan}\left(\frac{4}{3}\right)}$$

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i. $\int \frac{\arccos x}{\sqrt{1-x^2}} dx$

$u = \arccos x$
 $du = -\frac{1}{\sqrt{1-x^2}} dx$

$\int u du = \frac{1}{2}u^2 + C =$

$\frac{1}{2} \arccos^2 x + C$

j. $\int \frac{5}{3e^x - 2} dx \cdot \frac{e^{-x}}{e^{-x}} = \int \frac{5e^{-x}}{3 - 2e^{-x}} dx$

$u = 3 - 2e^{-x}$
 $du = 2e^{-x} dx$

$\frac{1}{2} du = e^{-x} dx$

$\frac{5}{2} \int \frac{du}{u} = \frac{5}{2} \ln u + C = \frac{5}{2} \ln |3 - 2e^{-x}| + C$

k. $\int \frac{1}{(x-1)\sqrt{4x^2 - 8x + 3}} dx = \int \frac{1}{(x-1)\sqrt{4x^2 - 8x + 4 - 1}} dx = \int \frac{1}{(x-1)\sqrt{(2x-2)^2 - 1}}$

$u = 2x - 2$ $x - 1 = \frac{1}{2}u$
 $du = 2 dx$
 $\frac{1}{2} du = dx$

$\int \frac{\frac{1}{2} du}{\frac{1}{2}u \sqrt{u^2 - 1}} = \operatorname{arcsec} u + C$

$= \operatorname{arcsec}(2x - 2) + C$

l. $\int 3^t dt = \frac{1}{\ln 3} 3^t + C$

3a. $f(x) = x^4 - 4x^2$ $g(x) = x^3 - 4x$

$x^4 - 4x^2 = x^3 - 4x$

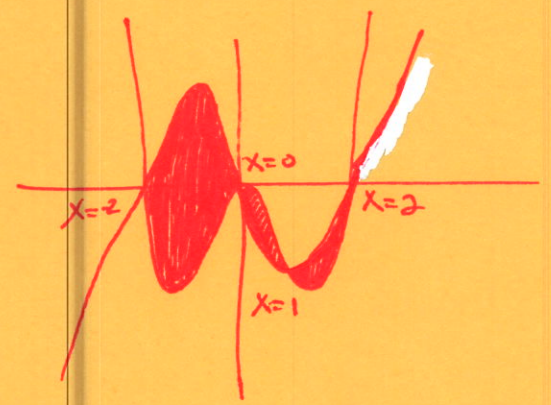
$x^4 - x^3 - 4x^2 + 4x = 0$

$x(x^3 - x^2 - 4x + 4) = 0$

$x[(x^2)(x-1) - 4(x-1)] = 0$

$x[(x-1)(x^2 - 4)] = 0$

$x = 0, x = 1, x = \pm 2$



$\int_{-2}^0 (x^3 - 4x) - (x^4 - 4x^2) dx + \int_0^1 (x^4 - 4x^2) - (x^3 - 4x) dx + \int_1^2 (x^3 - 4x) - (x^4 - 4x^2) dx$

$\int_{-2}^0 x^3 - 4x - x^4 + 4x^2 dx + \int_0^1 x^4 - 4x^2 - x^3 + 4x dx + \int_1^2 x^3 - 4x - x^4 + 4x^2 dx =$

$\left. \frac{1}{4}x^4 - \frac{4}{2}x^2 - \frac{1}{5}x^5 + \frac{4}{3}x^3 \right|_{-2}^0 + \left. \frac{1}{5}x^5 - \frac{4}{3}x^3 - \frac{1}{4}x^4 + \frac{4}{2}x^2 \right|_0^1 + \left. \frac{1}{4}x^4 - \frac{4}{2}x^2 - \frac{1}{5}x^5 + \frac{4}{3}x^3 \right|_1^2$

$\frac{124}{15} + \frac{37}{60} + \frac{53}{60} = \frac{293}{30}$

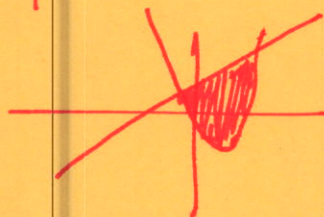
3b. $y = x^2 - 2x$, $y = x + 4$

$$x^2 - 2x = x + 4 \Rightarrow x^2 - 3x - 4 = 0$$

$$(x - 4)(x + 1) = 0$$

$$x = -1, x = 4$$

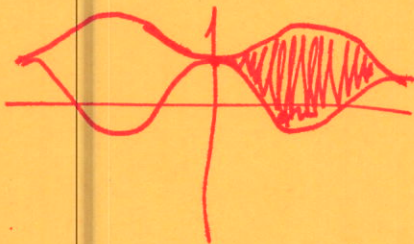
$$\int_{-1}^4 (x+4) - (x^2-2x) dx = \int_{-1}^4 -3x+4-x^2 dx = \left. \frac{3}{2}x^2 + 4x - \frac{1}{3}x^3 \right|_{-1}^4 = \frac{125}{6}$$



3c. $y = \cos x$, $y = 2 - \cos x$ $[0, 2\pi]$

$$\int_0^{2\pi} 2 - \cos x - \cos x dx = \int_0^{2\pi} 2 - 2\cos x dx$$

$$= 2x - 2\sin x \Big|_0^{2\pi} = 4\pi$$

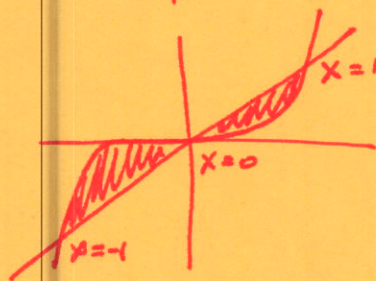


d. $y = x^3$, $y = x$

$$\int_{-1}^0 x^3 - x dx + \int_0^1 x - x^3 dx =$$

$$\left. \frac{1}{4}x^4 - \frac{1}{2}x^2 \right|_{-1}^0 + \left. \frac{1}{2}x^2 - \frac{1}{4}x^4 \right|_0^1 =$$

$$\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$



e. $y = |x|$, $y = x^2 - 2$

using symmetry

$$2 \int_0^2 x - (x^2 - 2) dx = 2 \int_0^2 x - x^2 + 2 dx =$$

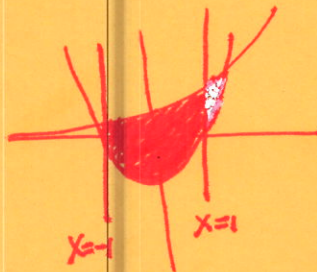
$$2 \left(\frac{1}{2}x^2 - \frac{1}{3}x^3 + 2x \right) \Big|_0^2 = 2 \left(\frac{10}{3} \right) = \frac{20}{3}$$



f. $y = e^x$, $y = x^2 - 1$, $x = -1$, $x = 1$

$$\int_{-1}^1 e^x - (x^2 - 1) dx = \left. e^x - \frac{1}{3}x^3 + x \right|_{-1}^1$$

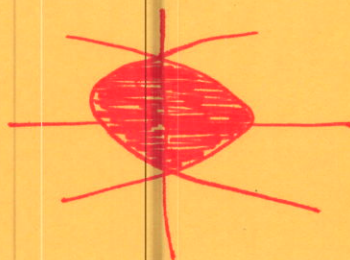
$$e - \frac{1}{3} + \frac{4}{3} \approx 3.6837$$



g. $x = 1 - y^2$, $x = y^2 - 1$

$$\int_{-1}^1 (1 - y^2) - (y^2 - 1) dy = \int_{-1}^1 -2y^2 + 2 dy$$

$$= \left. -\frac{2}{3}y^3 + 2y \right|_{-1}^1 = \frac{8}{3}$$



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3h. $X = y^4, y = \sqrt{2x}, y = 0$

$y = \sqrt[4]{x} \quad y^2 = 2x$

$(\sqrt[4]{x} = \sqrt{2x})^4 \Rightarrow x = 4x^2 \Rightarrow 4x^2 - x = 0$
 $x(4x - 1) = 0$
 $x = 0 \quad x = 1/4$

$\int_0^{1/4} \sqrt[4]{x} - \sqrt{2x} \, dx = \int_0^{1/4} x^{1/4} - \sqrt{2} \cdot x^{1/2} \, dx =$

$\frac{4}{5} x^{5/4} - \sqrt{2} \cdot \frac{2}{3} x^{3/2} \Big|_0^{1/4} = \frac{\sqrt{2}}{10} - \frac{\sqrt{2}}{12} = \frac{\sqrt{2}}{60} \approx .02357$

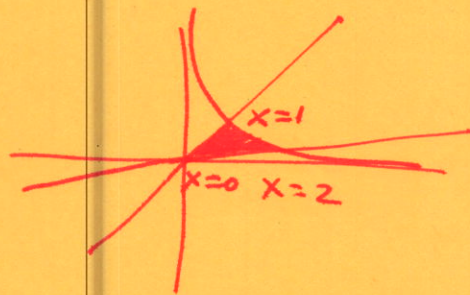


i. $y = 1/x, y = x, y = 1/4x$

$\int_0^1 x - 1/4x \, dx + \int_1^2 1/x - 1/4x \, dx$

$\int_0^1 3/4x \, dx + \int_1^2 \frac{1}{x} - \frac{x}{4} \, dx =$

$\frac{3}{8}x^2 \Big|_0^1 + \ln x - \frac{1}{8}x^2 \Big|_1^2 =$
 $\frac{3}{8} + \ln 2 - \frac{3}{8} = \ln 2$

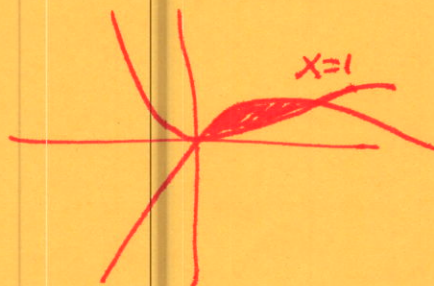


j. $y = x^2 e^{-x}, y = x e^{-x}$

$\int_0^1 x e^{-x} - x^2 e^{-x} \, dx = \int_0^1 (x - x^2) e^{-x} \, dx$

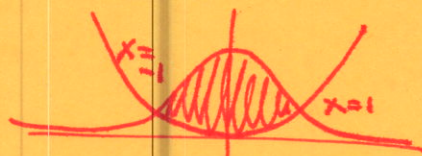
$+$	x	e^{-x}	dx
$-$	x^2	e^{-x}	dx
$+$	$-2x$	$-e^{-x}$	dx
$-$	-2	e^{-x}	dx
$-$	0	$-e^{-x}$	dx

$-(x - x^2) e^{-x} - (1 - 2x) e^{-x} + 2e^{-x} \Big|_0^1 =$
 $-(1 - 2)e^{-1} + (1)e^0 + 2e^{-1} - 2e^0 =$
 $\frac{1}{e} + 1 + \frac{2}{e} - 2 = \frac{3}{e} - 1$



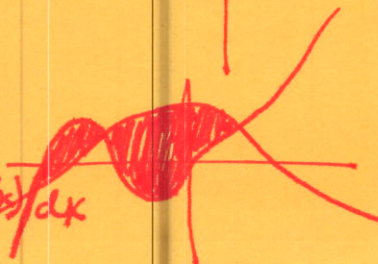
4a. $y = \frac{2}{1+x^4}, y = x^2$

$\int_1^2 \frac{2}{1+x^4} - x^2 \, dx$



b. $y = \cos x, y = x + 2 \sin^4 x$

$\int_{-1.224}^{-1.191} (x + 2 \sin^4 x) - \cos x \, dx + \int_{-1.224}^{.608} \cos x - (x + 2 \sin^4 x) \, dx$

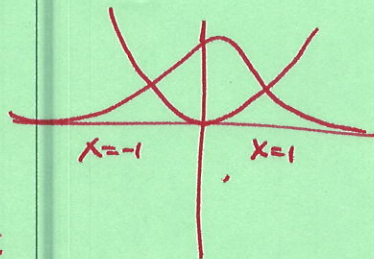


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4c. $y = e^{1-x^2}$ $y = x^4$

$$\int_{-1}^1 e^{1-x^2} - x^4 dx$$



5a. $\int x \cos 5x dx$

$u = x, dv = \cos 5x dx$
 $du = dx \quad v = \frac{1}{5} \sin 5x$

$$\frac{1}{5} x \sin 5x - \int \frac{1}{5} \sin 5x dx = \boxed{\frac{1}{5} x \sin 5x + \frac{1}{25} \cos 5x + C}$$

b. $\int \ln^2 x dx$

$u = \ln^2 x \quad dv = dx$
 $du = \frac{2 \ln x dx}{x} \quad v = x$

$x \ln^2 x - \int \frac{2 \ln x}{x} dx$

$u = \ln x \quad dv = dx$
 $du = \frac{1}{x} dx \quad v = x$

$$x \ln^2 x - 2x \ln x + 2 \int \frac{1}{x} dx =$$

$$\boxed{x \ln^2 x - 2x \ln x + 2x + C}$$

c. $\int t \cosh t dt$

$u = t \quad dv = \cosh t dt$
 $du = dt \quad v = \sinh t$

$t \sinh t - \int \cosh t dt =$

$$\boxed{t \sinh t - \cosh t + C}$$

d. $\int x^3 e^{-x^2} dx$

$u = x^2 \quad dv = x e^{-x^2}$
 $du = 2x dx \quad v = -\frac{1}{2} e^{-x^2}$

$\rightarrow w = -x^2 \quad dw = -2x dx \quad \int -\frac{1}{2} e^w dw$

$$-\frac{1}{2} x^2 e^{-x^2} + \int x e^{-x^2} dx = \boxed{-\frac{1}{2} x^2 e^{-x^2} - \frac{1}{2} e^{-x^2} + C}$$

e. $\int s 2^s ds$

$u = s \quad dv = 2^s ds$
 $du = ds \quad v = \frac{2^s}{\ln 2}$

$\frac{s 2^s}{\ln 2} - \int \frac{2^s}{\ln 2} ds =$

$$\boxed{\frac{s 2^s}{\ln 2} - \frac{1}{(\ln 2)^2} 2^s + C}$$

f. $\int \arctan 4t dt$

$u = \arctan 4t \quad dv = dt$
 $du = \frac{4}{1+16t^2} dt \quad v = t$

$t \arctan 4t - \int \frac{4t}{1+16t^2} dt$

$u = 1+16t^2$
 $du = 32t dt$

$$\boxed{t \arctan 4t - \frac{1}{8} \ln |1+16t^2| + C}$$

g. $\int (x^2+1)e^{-x} dx$

$u = (x^2+1) \quad dv = e^{-x}$
 $du = 2x \quad v = -e^{-x}$

$u = 2x \quad dv = e^{-x}$
 $du = 2 dx \quad v = -e^{-x}$

$$\boxed{-(x^2+1)e^{-x} - 2xe^{-x} - 2e^{-x} + C}$$

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5h. $\int_0^1 \frac{r^3}{\sqrt{4+r^2}} dr$

$u=r^2$
 $du=2r$

$dv = \frac{r}{\sqrt{4+r^2}}$
 $v = \sqrt{4+r^2}$

$w = 4+r^2$
 $dw = 2r dr$
 $v = \int \frac{1}{2} w^{-1/2} dw = \frac{1}{1/2} w^{1/2}$

$r^2 \sqrt{4+r^2} - \int 2r \sqrt{4+r^2} dr$

$r^2 \sqrt{4+r^2} - \frac{2}{3} (4+r^2)^{3/2} + C$

$r^2 \sqrt{4+r^2} - \frac{2}{3} (4+r^2)^{3/2} \Big|_0^1 =$

$(1) \sqrt{5} - \frac{2}{3} (5)^{3/2} - 0 + \frac{2}{3} (4)^{3/2} =$

$\sqrt{5} - \frac{10}{3} \sqrt{5} + \frac{2}{3} (8) = \sqrt{5} (-\frac{7}{3}) + \frac{16}{3} =$

$\frac{16-7\sqrt{5}}{3}$

i. $\int_0^\pi e^{\cos t} \sin 2t dt$

$2 \int_0^\pi e^{\cos t} \sin t \cos t dt \Rightarrow w = \cos t$
 $dw = -\sin t$

$-2 \int_1^{-1} e^w w dw = 2 \int_{-1}^1 w e^w dw$
 $u=w$
 $du=dw$
 $dv=e^w$
 $v=e^w$

$2 (w e^w - e^w) \Big|_{-1}^1 = 2 (e - e - (-1e^{-1} - e^{-1})) = 2 (0 + \frac{1}{e} + \frac{1}{e}) = 2 (\frac{2}{e}) = \frac{4}{e}$

6. $\int x^n \ln x dx$

$u = \ln x$
 $du = \frac{1}{x}$

$dv = x^n dx$
 $v = \frac{1}{(n+1)} x^{n+1}$

$\frac{1}{(n+1)} x^{n+1} \ln x - \int \frac{1}{(n+1)} x^{n+1} \cdot \frac{1}{x} dx$

$= \frac{1}{(n+1)} x^{n+1} \ln x - \frac{1}{n+1} \int x^n dx = \left[\frac{1}{(n+1)} x^{n+1} \ln x - \frac{1}{(n+1)^2} x^{n+1} + C \right]$

7. a.

\pm	u	dv
+	z^3	e^z
-	$3z^2$	e^z
+	$6z$	e^z
-	6	e^z
+	0	e^z

$\int z^3 e^z dz = z^3 e^z - 3z^2 e^z + 6z e^z - 6e^z + C$

b.

\pm	u	dv
+	t^4	$\sin ht$
-	$4t^3$	$\cos ht$
+	$12t^2$	$\sin ht$
-	$24t$	$\cos ht$
+	24	$\sin ht$
-	0	$\cos ht$

$\int t^4 \sin ht dt =$
 $t^4 \cos ht - 4t^3 \sin ht +$
 $12t^2 \cos ht - 24t \sin ht +$
 $24 \cos ht + C$

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7c.	+	u	dv
	+	p^5	$\cos p$
	-	$5p^4$	$\sin p$
	+	$20p^3$	$-\cos p$
	-	$60p^2$	$-\sin p$
	+	$120p$	$\cos p$
	-	120	$\sin p$
	+	0	$-\cos p$

$$\int p^5 \cos p \, dp = p^5 \sin p + 5p^4 \cos p - 20p^3 \sin p - 60p^2 \cos p + 120p \sin p + 120 \cos p + C$$

8a. $\int e^{2\theta} \sin 3\theta \, d\theta$

$$u = \sin 3\theta \\ du = 3 \cos 3\theta$$

$$dv = e^{2\theta} d\theta$$

$$v = \frac{1}{2} e^{2\theta}$$

$$\frac{1}{2} \sin 3\theta \cdot e^{2\theta} - \frac{3}{2} \int \cos 3\theta \cdot e^{2\theta} \cdot d\theta$$

$$u = \cos 3\theta \quad dv = e^{2\theta} d\theta \\ du = -3 \sin 3\theta d\theta \quad v = \frac{1}{2} e^{2\theta}$$

$$\frac{1}{2} e^{2\theta} \sin 3\theta - \frac{3}{2} \left[\frac{1}{2} \cos 3\theta \cdot e^{2\theta} + \int + \frac{3}{2} \sin 3\theta e^{2\theta} d\theta \right]$$

$$\frac{1}{2} e^{2\theta} \sin 3\theta - \frac{3}{4} e^{2\theta} \cos 3\theta - \frac{9}{4} \int e^{2\theta} \sin 3\theta \, d\theta = \int e^{2\theta} \sin 3\theta \, d\theta + \frac{9}{4} \int e^{2\theta} \sin 3\theta \, d\theta = \frac{9}{4} \int e^{2\theta} \sin 3\theta \, d\theta$$

$$\frac{1}{2} e^{2\theta} \sin 3\theta - \frac{3}{4} e^{2\theta} \cos 3\theta = \frac{13}{4} \int e^{2\theta} \sin 3\theta \, d\theta$$

$$\frac{4}{13} \left[\frac{1}{2} e^{2\theta} \sin 3\theta - \frac{3}{4} e^{2\theta} \cos 3\theta \right] + C = \int e^{2\theta} \sin 3\theta \, d\theta$$

$$= \boxed{\frac{2}{13} e^{2\theta} \sin 3\theta - \frac{3}{13} e^{2\theta} \cos 3\theta + C}$$

b. $\int \sec^3 \theta \, d\theta$

$$u = \sec \theta \quad dv = \sec^2 \theta \\ du = \sec \theta \tan \theta \quad v = \tan \theta$$

$$\sec \theta \tan \theta - \int \sec \theta \tan^2 \theta \, d\theta = \sec \theta \tan \theta - \int \sec \theta (\sec^2 \theta - 1) \, d\theta =$$

$$\sec \theta \tan \theta - \int \sec^3 \theta \, d\theta + \int \sec \theta \, d\theta = \int \sec^3 \theta \, d\theta + \int \sec^3 \theta \, d\theta$$

$$\sec \theta \tan \theta + \ln |\sec \theta + \tan \theta| = 2 \int \sec^3 \theta \, d\theta$$

So conti'd

$$\int \sec^3 \theta d\theta = \frac{1}{2} \sec \theta \tan \theta + \frac{1}{2} \ln |\sec \theta + \tan \theta| + C$$

$$9. \int e^{2\theta} \sin 3\theta d\theta = \int e^{2\theta} \left(\frac{e^{3i\theta} - e^{-3i\theta}}{2i} \right) d\theta =$$

$$\frac{1}{2i} \int e^{(2+3i)\theta} - e^{(2-3i)\theta} d\theta = \frac{1}{2i} \left[\frac{e^{(2+3i)\theta}}{2+3i} - \frac{e^{(2-3i)\theta}}{2-3i} \right] + C =$$

$$\frac{1}{2i} \left[\frac{e^{(2+3i)\theta} (2-3i)}{(2+3i)(2-3i)} - \frac{e^{(2-3i)\theta} (2+3i)}{(2-3i)(2+3i)} \right] + C =$$

$$\frac{1}{2i} \left[\frac{2e^{(2+3i)\theta} - 3ie^{(2+3i)\theta} - 2e^{(2-3i)\theta} - 3ie^{(2-3i)\theta}}{4+9} \right] + C =$$

$$\frac{1}{2i} \cdot \frac{1}{13} e^{2\theta} \left[\frac{2e^{3i\theta} - 2e^{-3i\theta} - 3ie^{3i\theta} - 3ie^{-3i\theta}}{2i} \right] + C =$$

$$\frac{1}{13} e^{2\theta} \left[2 \frac{(\sin 3\theta)}{\frac{e^{3i\theta} - e^{-3i\theta}}{2}} - 3 \frac{(\cos 3\theta)}{\frac{e^{3i\theta} + e^{-3i\theta}}{2i}} \right] + C =$$

$$\boxed{\frac{2}{13} e^{2\theta} \sin 3\theta - \frac{3}{13} e^{2\theta} \cos 3\theta + C}$$

matches 8a.