192 Homework #11 Key

1.a. $\sum_{n=1}^{\infty} \frac{2n}{n+1}$ diverges by nth term test $\lim_{n\to\infty} \frac{2n}{n+1} = 2 \neq 0$

b. $\sum_{n=1}^{\infty} \frac{n7^n}{n!}$ $\lim_{n\to\infty} \frac{\cancel{(n+1)}7^{\cancel{n+1}}}{\cancel{(n+1)!}} \cdot \frac{\cancel{n!}}{\cancel{n7^n}} = \lim_{n\to\infty} \frac{7}{n} = 0 < 1$

Converges by ratio test

c. $\sum_{n=1}^{\infty} \frac{(-1)^n 3^{n-2}}{2^n} = \sum_{n=1}^{\infty} \frac{1}{9}\left(\frac{-3}{2}\right)^n$ diverges by geometric series

d. $\sum_{n=1}^{\infty} \frac{\ln n}{n^2} < \sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2} = \sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ converges by p-series

Converges by direct comparison

2a. $\sum_{n=1}^{\infty} \frac{(-1)^n (x+1)^n}{n}$ $\lim_{n\to\infty} \left| \frac{(x+1)^{n+1}}{n+1} \cdot \frac{n}{(x+1)^n} \right| = \lim_{n\to\infty} x+1 < 1$

$$\begin{array}{ccccc} -1 < & x+1 & < 1 \\ -1 & -1 & -1 \end{array} \Rightarrow R = 1, \quad -2 < x < 0$$

$x=-2$ $\sum_{n=1}^{\infty} \frac{(-1)^n(-1)^n}{n} = \sum_{n=1}^{\infty} \frac{1}{n}$ diverges by harmonic series
p-series test

$x=0$ $\sum_{n=1}^{\infty} \frac{(-1)^n(1)^n}{n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ Converges by ALT $\lim_{n\to\infty} \frac{1}{n} = 0$

interval $(-2, 0]$

b. $\sum_{n=1}^{\infty} \frac{(x+1)^n}{n!}$ $\lim_{n\to\infty} \left| \frac{(x+1)^{n+1}}{(n+1)!} \cdot \frac{n!}{(x+1)^n} \right| = \lim_{n\to\infty} \frac{x+1}{n+1} = 0 < 1$

$R = \infty$, interval $(-\infty, \infty)$ converges for all x

c. $\sum_{n=0}^{\infty} 2\left(\frac{x}{8}\right)^{3n} = \sum_{n=0}^{\infty} 2\left(\frac{x^3}{512}\right)^n$ $\left|\frac{x^3}{512}\right| < 1 \Rightarrow -1 < \frac{x^3}{512} < 1$

$-512 < x^3 < 512 \Rightarrow -8 < x < 8$ $R = 8$

$(-\infty, \infty)$

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3a. $\sum_{n=0}^{\infty} \left(\frac{x}{2}\right)^n$ $\quad \left|\frac{x}{2}\right| < 1 \Rightarrow -1 < \frac{x}{2} < 1 \Rightarrow -2 < x < 2 \quad (-2,2)$

b. $\sum_{n=0}^{\infty} \frac{(-1)^n n!(x-4)^n}{3^n}$ $\quad \lim_{n\to\infty} \left|\frac{(n+1)!(x-4)^{n+1}}{3^{n+1}} \cdot \frac{3^n}{n!(x-4)^n}\right| = \lim_{n\to\infty} \left|\frac{(n+1)(x-4)}{3}\right|$

$= \infty$ for all x except $x=4$ (where series is always 0

$R=0$, converges $x=4$

c. $\sum_{n=1}^{\infty} \frac{n(-2x)^{n-1}}{n+1}$ $\quad \lim_{n\to\infty} \left|\frac{(n+1)(-2x)^n}{n+2} \cdot \frac{n+1}{n(-2x)^{n-1}}\right| = \lim_{n\to\infty} |-2x| < 1$

$-1 < 2x < 1 \Rightarrow -\frac{1}{2} < x < \frac{1}{2} \quad (-1/2, 1/2)$

diverges at endpoints

d. $\sum_{n=1}^{\infty} \frac{(-3)^n x^n}{n\sqrt{n}}$ $\quad \lim_{n\to\infty} \left|\frac{(-3)^{n+1}x^{n+1}}{(n+1)\sqrt{n+1}} \cdot \frac{(n)\sqrt{n}}{(-3)^n x^n}\right| = \lim_{n\to\infty} |-3x| < 1$

$-1 < 3x < 1 \Rightarrow -\frac{1}{3} < x < 1/3$

$x = -1/3 \quad \sum_{n=1}^{\infty} \frac{(-3)^n(-1/3)^n}{n\sqrt{n}} = \sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ converges as a p-series

$x = 1/3 \quad \sum_{n=1}^{\infty} \frac{(-3)^n(1/3)^n}{n\sqrt{n}} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n^{3/2}}$ converges by ALT

$[-1/3, 1/3]$

e. $\sum_{n=1}^{\infty} \frac{x^n}{n3^n}$ $\quad \lim_{n\to\infty} \left|\frac{x^{n+1}}{(n+1)3^{n+1}} \cdot \frac{n3^n}{x^n}\right| = \lim_{n\to\infty} \left|\frac{x}{3}\right| < 1 \quad -1 < \frac{x}{3} < 1$

$\Rightarrow -3 < x < 3$

$x = -3 \quad \sum_{n=1}^{\infty} \frac{(-3)^n}{n3^n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ converges by ALT

$x = 3 \quad \sum_{n=1}^{\infty} \frac{3^n}{n3^n} = \sum_{n=1}^{\infty} \frac{1}{n}$ diverges by p-series

$[-3, 3)$

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3f. $\sum_{n=1}^{\infty} n!\,(2x-1)^n \quad \lim_{n\to\infty}\left|\frac{(n+1)!\,(2x-1)^{n+1}}{n!\,(2x-1)^n}\right| = \lim_{n\to\infty}\left|(n+1)(2x-1)\right| = \infty$

diverges for all values except $x = 1/2$

g. $\sum_{n=2}^{\infty} \frac{x^{2n}}{n(\ln n)^2} \quad \lim_{n\to\infty}\left|\frac{x^{2n+2}}{(n+1)\ln^2(n+1)}\cdot\frac{n\ln^2 n}{x^{2n}}\right| = \lim_{n\to\infty} x^2 < 1$

$-1 < x < 1$ $\qquad$ 1, −1 both behave the same

$x = 1, -1 \quad \sum_{n=2}^{\infty} \frac{1^{2n}}{n\ln^2 n} = \sum_{n=2}^{\infty} \frac{1}{n\ln^2 n}$ converges by p-series/integral test

h. $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{(n+1)(n+2)} \quad \lim_{n\to\infty}\left|\frac{x^{n+1}}{(n+2)(n+3)}\cdot\frac{(n+1)(n+2)}{x^n}\right| = \lim_{n\to\infty}|x| < 1$

$-1 < x < 1$

$x = -1 \quad \sum_{n=1}^{\infty} \frac{(-1)^n(-1)^n}{(n+1)(n+2)}$ converges by direct comparison w/ $\sum_{n=1}^{\infty}\frac{1}{n^2}$

$x = 1 \quad \sum_{n=1}^{\infty} \frac{(-1)^n 1^n}{(n+1)(n+2)}$ converges by AST $\lim_{n\to\infty}\frac{1}{(n+1)(n+2)} = 0$

i. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{n!} \quad \lim_{n\to\infty}\left|\frac{x^{2n+2}}{(n+1)!}\cdot\frac{n!}{x^{2n}}\right| = \lim_{n\to\infty}\left|\frac{x^2}{n+1}\right| < 0$

converges for all x $\quad (-\infty, \infty)$

j. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{n!\,(n+1)!\,2^{2n+1}} \quad \lim_{n\to\infty}\left|\frac{x^{2n+3}}{\cancel{(n+1)!}\,(n+2)!\,2^{2n+3}}\cdot\frac{n!\,\cancel{(n+1)!}\,2^{2n+1}}{x^{2n+1}}\right| = \lim_{n\to\infty}\left|\frac{x^2}{(n+1)(n+2)2^2}\right|$

$= 0 < 1$ converges for all x $\quad (-\infty, \infty)$

k. $\sum_{n=1}^{\infty} \frac{3^n(x+4)^n}{\sqrt{n}} \quad \lim_{n\to\infty}\left|\frac{3^{n+1}(x+4)^{n+1}}{\sqrt{n+1}}\cdot\frac{\sqrt{n}}{3^n(x+4)^n}\right| = \lim_{n\to\infty} 3(x+4) < 1$

$-1 < 3(x+4) < 1 \Rightarrow -\frac{1}{3} < x+4 < \frac{1}{3}$
$\quad -4 \qquad -4 \qquad -4$

$\Rightarrow -\frac{13}{3} < x < -\frac{11}{3}$

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3k cont'd

$x = -13/3$ $\displaystyle\sum_{n=1}^{\infty} \frac{3^n(-\frac{13}{3}+4)^n}{\sqrt{n}} = \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ converges by ALT

$x = -11/3$ $\displaystyle\sum_{n=1}^{\infty} \frac{3^n(-11/3+4)^n}{\sqrt{n}} = \sum_{n=1}^{\infty} \frac{1^n}{\sqrt{n}}$ diverges by p-series

$[-13/3, -11/3)$

l. $\displaystyle\sum_{n=1}^{\infty} \frac{(2x-1)^n}{5^n\sqrt{n}}$ $\displaystyle\lim_{n\to\infty} \left| \frac{(2x-1)^{n+1}}{5^{n+1}\sqrt{n+1}} \cdot \frac{5^n\sqrt{n}}{(2x-1)^n} \right| = \lim_{n\to\infty} \left| \frac{2x-1}{5} \right| < 1$

$-1 < \frac{2x-1}{5} < 1 \Rightarrow \underset{+1}{-5} < \underset{+1}{2x-1} < \underset{+1}{5} \Rightarrow \frac{-4}{2} < \frac{2x}{2} < \frac{6}{2}$

$\Rightarrow -2 < x < 3$

$x = -2$ $\displaystyle\sum_{n=1}^{\infty} \frac{(-5)^n}{5^n\sqrt{n}} = \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ Converges by ALT

$x = 3$ $\displaystyle\sum_{n=1}^{\infty} \frac{5^n}{5^n\sqrt{n}} = \sum_{n=1}^{\infty} \frac{1^n}{\sqrt{n}}$ diverges by p-series

$[-2, 3)$

m. $\displaystyle\sum_{n=2}^{\infty} \frac{5^n}{\ln n}(x-3)^n$ $\displaystyle\lim_{n\to\infty} \left| \frac{5^{n+1}(x-3)^{n+1}}{\ln(n+1)} \cdot \frac{\ln n}{5^n(x-3)^n} \right| = \lim_{n\to\infty} |5(x-3)| < 1$

$-1 < 5(x-3) < 1 \Rightarrow \underset{+3}{-1/5} < \underset{+3}{x-3} < \underset{+3}{1/5} \Rightarrow \frac{14}{5} < x < \frac{16}{5}$

$x = \frac{14}{5}$ $\displaystyle\sum_{n=2}^{\infty} \frac{5^n(\frac{14}{5}-3)^n}{\ln n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{\ln n}$ Converges by ALT $\displaystyle\lim_{n\to\infty} \frac{1}{\ln n} = 0$

$x = \frac{16}{5}$ $\displaystyle\sum_{n=2}^{\infty} \frac{5^n(\frac{16}{5}-3)^n}{\ln n} = \sum_{n=1}^{\infty} \frac{1^n}{\ln n}$ diverges by direct comparison w/ $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$ harmonic series

$[14/5, 16/5)$

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4a. $f(x) = \frac{1}{2-x} \cdot \frac{1/2}{1/2} = \frac{1/2}{1-\frac{x}{2}}$ $\quad a=\frac{1}{2}, r=\frac{x}{2}$

$= \sum_{n=0}^{\infty} \frac{1}{2}\left(\frac{x}{2}\right)^n = \sum_{n=0}^{\infty} \frac{x^n}{2^{n+1}}$

b. $f(x) = \frac{3}{2x-1}$ $\quad c=2$ $\qquad 2x-1 = \underset{2x-4}{2(x-2)}-1+4 = 2(x-2)+3$

$= \frac{3}{3+2(x-2)} \cdot \frac{1/3}{1/3} = \frac{1}{1+\frac{2}{3}(x-2)}$ $\quad a=1, r=\left(\frac{2}{3}(x-2)\right)$

$= \sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^n (x-2)^n$

c. $f(x) = \frac{1}{1-x^2}$ $\quad a=1, r=x^2$ $\qquad \sum_{n=0}^{\infty} x^{2n}$

d. $f(x) = \frac{x}{(1-x)^2}$ $\quad a=x, r=x$ derivative $\sum_{n=0}^{\infty} a(n+1)x^n$

$= \sum_{n=0}^{\infty} x(n+1)x^n = \sum_{n=0}^{\infty} (n+1)x^{n+1}$

e. $f(x) = $ arctan x $\qquad f'(x) = \frac{1}{1+x^2}$ $\quad a=1, r=-x^2$

$\sum_{n=0}^{\infty} (-x^2)^n = \sum_{n=0}^{\infty} (-1)^n x^{2n} = f'(x)$ $\qquad f(x) = \int f'(x)\,dx = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1} + c$

$\arctan(0) = 0 \Rightarrow c = 0$

$= \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}$

f. $f(x) = \frac{3}{2x-1} = \frac{-3}{1-2x}$ $\quad a=-3, r=2x$ $\qquad \sum_{n=0}^{\infty} -3(2x)^n = \sum_{n=0}^{\infty} -3 \cdot 2^n x^n$

g. $f(x) = \frac{3x}{x^2+x-2} = \frac{3x}{x^2+x+1/4-\frac{9}{4}} = \frac{3x}{(x+1/2)^2-\frac{9}{4}} = \frac{3(x+1/2)-\frac{3}{2}}{(x+1/2)^2-9/4} \cdot \frac{-\frac{4}{9}}{-\frac{4}{9}}$

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4g cont'd $= \dfrac{-\frac{4}{3}(x+\frac{1}{2})+\frac{2}{3}}{1-\frac{4}{9}(x+\frac{1}{2})^2} = \dfrac{-\frac{4}{3}(x+\frac{1}{2})}{1-\frac{4}{9}(x+\frac{1}{2})^2} + \dfrac{\frac{2}{3}}{1-\frac{4}{9}(x+\frac{1}{2})^2}$

$a = -\frac{4}{3}(x+\frac{1}{2})$ $\quad a = \frac{2}{3}$

$r = \frac{4}{9}(x+\frac{1}{2})^2$ $\quad r = \frac{4}{9}(x+\frac{1}{2})^2$

$$= \sum_{n=0}^{\infty} -\frac{4}{3}(x+\tfrac{1}{2})\left(\frac{4}{9}\right)^n (x+\tfrac{1}{2})^{2n} + \sum_{n=0}^{\infty} \frac{2}{3}\left(\frac{4}{9}\right)^n (x+\tfrac{1}{2})^{2n}$$

$$= \sum_{n=0}^{\infty} -\frac{4^{n+1}}{3^{2n+1}}(x+\tfrac{1}{2})^{2n+1} + \sum_{n=0}^{\infty} \frac{2^{2n+1}}{3^{2n+1}}(x+\tfrac{1}{2})^{2n}$$

h. $f(x) = \ln(x+1)$ $\quad f'(x) = \dfrac{1}{1+x}$ $\quad a=1,\ r=-x$

$f'(x) = \sum_{n=0}^{\infty} (-1)^n x^n$ $\quad f(x) = \int \sum_{n=0}^{\infty} (-1)^n x^n \, dx = \sum_{n=0}^{\infty} \dfrac{(-1)^n x^{n+1}}{n+1} + C$

$\ln(1+0) = 0 \Rightarrow C = 0$

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{n+1}$$

i. $f(x) = \dfrac{7x^3}{(1+3x^2)^4}$ $\quad a(1-r)^{-1} = \sum_{n=0}^{\infty} ax^n$

$a(1-r)^{-2} = \sum_{n=1}^{\infty} anx^{n-1} = \sum_{n=0}^{\infty} a(n+1)x^n$

$2a(1-r)^{-3} = \sum_{n=2}^{\infty} an(n-1)x^{n-2} = \sum_{n=0}^{\infty} a(n+2)(n+1)x^n$

$6a(1-r)^{-4} = \sum_{n=3}^{\infty} an(n-1)(n-2)x^{n-3} = \sum_{n=0}^{\infty} a(n+3)(n+2)(n+1)x^n$

$\dfrac{6a}{(1-r)^4}$

$a = \frac{7}{6}x^3$

$r = (-3x^2)$ $\quad \sum_{n=0}^{\infty} \frac{7}{6}x^3(n+3)(n+2)(n+1)(-3x^2)^n = \sum_{n=0}^{\infty} \frac{7}{6}(-3)^n(n+3)(n+2)(n+1)x^{2n+3}$

$$= \sum_{n=0}^{\infty} \frac{7}{6}(-3)^n(n+3)(n+2)(n+1)x^{2n+3}$$

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5a. $f(x) = \frac{1}{1+x^5}$ $a=1$, $r=-x^5$ $\sum_{n=0}^{\infty} (-x^5)^n = \sum_{n=0}^{\infty} (-1)^n x^{5n}$

$$\int_0^{.2} \sum_{n=0}^{\infty} (-1)^n x^{5n}\, dx = \sum_{n=0}^{\infty} \frac{(-1)^n x^{5n+1}}{5n+1} \Big|_0^{.2} = \sum_{n=0}^{\infty} \frac{(-1)^n (.2)^{5n+1}}{5n+1}$$

$n=0$ $\frac{(-1)^0 (.2)^1}{1}$ $n=1$ $\frac{(-1)^1 (.2)^6}{6}$ $n=2$ $\frac{(-1)^2 (.2)^{11}}{11}$

$= .2$ $\quad - 1.067 \times 10^{-5}$ $\quad + \quad 1.86 \times 10^{-9}$

error < 6 decimals

$= 0.199989\overline{33}$

b. $x \arctan 3x$

$f(x) = \arctan 3x$

$f'(x) = \frac{3}{1+9x^2}$ $a=3$, $r=-9x^2$

$$\sum_{n=0}^{\infty} 3(-9x^2)^n = \sum_{n=0}^{\infty} (-1)^n 3^{2n+1} x^{2n+1}$$

$$f(x) = \sum_{n=0}^{\infty} \int (-1)^n 3^{2n+1} x^{2n+1}\, dx = \sum_{n=0}^{\infty} \frac{(-1)^n 3^{2n+1} x^{2n+2}}{2n+2}$$

$$x \arctan 3x = x \sum_{n=0}^{\infty} \frac{(-1)^n 3^{2n+1} x^{2n+2}}{2n+2} = \sum_{n=0}^{\infty} \frac{(-1)^n 3^{2n+1} x^{2n+3}}{2n+2}$$

$$\int_0^{.1} \sum_{n=0}^{\infty} \frac{(-1)^n 3^{2n+1} x^{2n+3}}{2n+2}\, dx = \sum_{n=0}^{\infty} \frac{(-1)^n 3^{2n+1} x^{2n+4}}{(2n+2)(2n+4)} \Big|_0^{.1} =$$

$$\sum_{n=0}^{\infty} \frac{(-1)^n 3^{2n+1} (.1)^{2n+4}}{(2n+2)(2n+4)}$$

$n=0$ $\frac{(-1)^0 3 (.1)^4}{2 \cdot 4} = 3.75 \times 10^{-5}$ $\quad n=1$ $\frac{(-1)^1 3^3 (.1)^6}{4 \cdot 6} = -1.1255 \times 10^{-6}$

$n=2$ $\frac{(-1)^2 3^5 (.1)^8}{6 \cdot 8} = 5.0625 \times 10^{-8}$ $\quad$ Sum 3.6375×10^{-5}

error

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6. $\sum_{n=0}^{\infty} \frac{n^n}{(2n)!}$ $\quad \lim_{n\to\infty} \frac{(n+1)^{n+1}}{(2n+2)!}\cdot\frac{(2n)!}{n^n} = \lim_{n\to\infty}\left|\frac{(n+1)^n}{n^n}\cdot\frac{(n+1)}{(2n+1)(2n+2)}\right| =$

$$\underbrace{\lim_{n\to\infty}\left|\frac{n+1}{n}\right|^n}_{=e}\cdot\underbrace{\lim_{n\to\infty}\left|\frac{n+1}{(2n+1)(2n+2)}\right|}_{=0} = 0 \qquad (2n)! >> n^n$$

$\sum_{n=0}^{\infty} \frac{(n!)^2}{(2n)!}$ $\quad \lim_{n\to\infty}\left|\frac{(n+1)!(n+1)!}{(2n+2)!}\cdot\frac{(2n)!}{n!\,n!}\right| = \lim_{n\to\infty}\left|\frac{(n+1)(n+1)}{(2n+2)(2n+3)}\right| = \frac{1}{4}$

$$(n!)^2 << (2n)!$$

$\sum_{n=0}^{\infty} \frac{n^n}{(n!)^2}$ $\quad \lim_{n\to\infty}\left|\frac{(n+1)^{n+1}}{(n+1)!(n+1)!}\cdot\frac{n!\,n!}{n^n}\right| = \lim_{n\to\infty}\left|\frac{(n+1)^{n\cancel{+1}}}{(n+1)\cancel{(n+1)}\cdot n^n}\right| =$

$$\lim_{n\to\infty}\underbrace{\left(\frac{n+1}{n}\right)^n}_{=e}\cdot\frac{1}{n} = 0 \qquad (n!)^2 >> n^n$$

$$n^n << (n!)^2 << (2n)!$$

$\sum_{n=0}^{\infty} \frac{n^{2n}}{(2n)!}$ $\quad \lim_{n\to\infty}\left|\frac{(n+1)^{2n+2}}{(2n+2)!}\cdot\frac{(2n)!}{n^{2n}}\right| = \lim_{n\to\infty}\underbrace{\left|\frac{n+1}{n}\right|^{2n}}_{e^2}\cdot\frac{(n+1)^2}{(2n+2)(2n+1)} = \frac{e^2}{4}$

$$n^n << (n!)^2 << (2n)! << n^{2n}$$

$\sum_{n=0}^{\infty} \frac{n^{2n}}{a^{n^2}}$ $\quad \lim_{n\to\infty}\left|\frac{(n+1)^{2n+2}}{a^{\cancel{n^2}+2n+1}}\cdot\frac{\cancel{a^{n^2}}}{n^{2n}}\right| = \lim_{n\to\infty}\underbrace{\left|\frac{n+1}{n}\right|^{2n}}_{e^2}\cdot\left|\frac{1}{a^{2n+1}}\right| = 0$

$$n^n << (n!)^2 << (2n)! << n^{2n} << a^{n^2}$$

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6 cont'd

$$\sum_{n=0}^{\infty} \frac{a^{n^2}}{(n^2)!} \quad \lim_{n\to\infty} \left| \frac{\cancel{a^{n^2}}a^{2n+1}}{(n^2+2n+1)!} \cdot \frac{(n^2)!}{\cancel{a^{n^2}}} \right| = \lim_{n\to\infty} \left| \frac{a^{2n+1}}{(n^2+1)(n^2+2)\cdots(n^2+2n)(n^2+2n+1)} \right| = 0$$

$$n^n << (n!)^2 << (2n)! << n^{2n} << a^{n^2} << (n^2)!$$

$$\sum_{n=0}^{\infty} \frac{(n^2)!}{n^{n^2}} \quad \lim_{n\to\infty} \left| \frac{(n^2+2n+1)!}{(n+1)^{\cancel{n^2}+2n+1}} \cdot \frac{\cancel{n^{n^2}}}{(n^2)!} \right| = \lim_{n\to\infty} \left| \frac{(n^2+1)(n^2+2)+\cdots(n^2+2n)(n^2+2n+1)}{(n+1)^{2n+1}} \right|$$

$$(n^2+1)(n^2+2)\cdots(n^2+2n)(n^2+2n+1) > (n^2+1)^{2n+1}$$

$$\approx \lim_{n\to\infty} \left| \frac{(n^2+1)^{2n+1}}{(n+1)^{2n+1}} \right| = \lim_{n\to\infty} \left| \frac{n^2+1}{n+1} \right|^{2n+1} = \infty$$

$$\ln^a n << n^a << n^a \ln^b n << a^n << n! << n^n << (n!)^2 << (2n)! << n^{2n}$$

$$<< a^{n^2} << n^{n^2} << (n^2)!$$