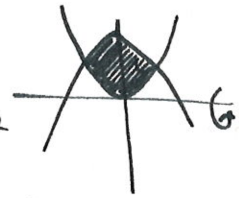
MAT 192, Final Exam, Fall 2016 Name KEY

Instructions: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Find the volume of the solid of revolution defined by $y = \frac{1}{4}x^2, y = 5 - x^2$ revolved around the x-axis. (6 points)

$r = 5 - x^2 - \frac{1}{4}x^2 = 5 - \frac{5}{4}x^2 = 5(1 - \frac{1}{4}x^2)$

$= 0 \Rightarrow 1 = \frac{1}{4}x^2 \Rightarrow 4 = x^2 \Rightarrow x = \pm 2$ $\quad r_i = \frac{1}{4}x^2 \quad r_o = 5 - x^2$



$\pi \int_{-2}^{2} -(\frac{1}{4}x^2)^2 + (5 - x^2)^2 \, dx = 2\pi \int_0^2 -\frac{1}{16}x^4 + (25 - 10x^2 + x^4) \, dx$

$2\pi \int_0^2 -10x^2 + 25 + \frac{15}{16}x^4 \, dx = 2\pi \left[-\frac{10}{3}x^3 + 25x + \frac{3}{16}x^5 \right|_0^2 =$

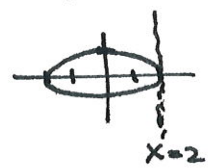
$2\pi \left[-\frac{80}{3} + 50 + 6 \right] = 2\pi \left[\frac{88}{3} \right] = \frac{176\pi}{3}$

2. Set up an integral to find the volume of the solid of revolution defined by $x^2 + 4y^2 = 4$, revolved around the line $x = 2$. You do not need to integrate it. (5 points)

$4y^2 = \frac{4 - x^2}{4}$

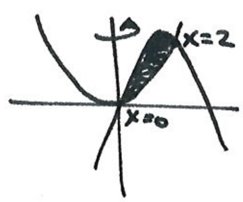
$y = \pm \frac{\sqrt{4 - x^2}}{2}$

$r = \frac{\sqrt{4-x^2}}{2} - \left(-\frac{\sqrt{4-x^2}}{2} \right) = \sqrt{4 - x^2}$



$2\pi \int_{-2}^{2} \sqrt{4 - x^2} \, (2 - x) \, dx$ (using shells)

3. Find the volume of the solid of revolution bounded by $y = x^2, y = 6x - 2x^2$ around the y-axis. (6 points)

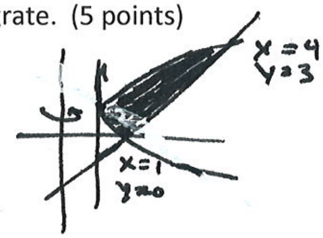


$2\pi \int_0^2 (6x - 2x^2 - x^2) x \, dx$

$2\pi \int_0^2 6x^2 - 3x^3 \, dx = 2\pi \left[2x^3 - \frac{3}{4}x^4 \right|_0^2 =$

$2\pi [16 - 12] = 8\pi$

4. Set up an integral to find the volume of the solid of revolution defined by $x = (y-1)^2$, $x - y = 1$, revolved around the line $x = -1$. You do not need to integrate. (5 points)



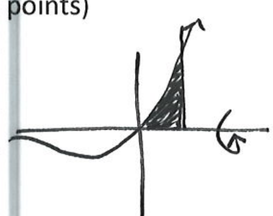
$x = y+1$

$r_o = y+1+1 = \quad y+2$

$r_i = (y-1)^2+1$

$$\pi \int_0^3 (y+2)^2 - [(y-1)^2+1]^2 \, dy$$

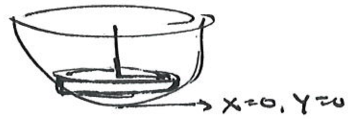
5. Set up an integral to find the surface area of the surface of the surface of revolution defined by $y = xe^x$ on the interval $[0,1]$ revolved around the x-axis. (5 points)



$y' = e^x + xe^x = e^x(1+x)$

$$2\pi \int_0^1 xe^x \sqrt{1 + e^{2x}(1+x)^2} \, dx$$

6. A spherical tank with a radius of 3 meters is full of water. Find the work done pumping all the water out of the top of the tank. Recall that the density of water is 1000 kg/m^3. (7 points)



$x^2+y^2=r^2$

$$1000 \int_0^6 \pi(9-(y-3)^2)(6-y)\,dy$$

$$1000\pi \int_0^6 (\cancel{9} - (\cancel{9} - 6y + y^2))(6-y)\,dy$$

$$1000\pi \int_0^6 (6y-y^2)(6-y)\,dy = 1000\pi \int_0^6 36y - 6y^2 - 6y^2 + y^3 \, dy$$

$-12y^2$

$x^2 + (y-3)^2 = 9$

$(y-3)^2 = 9 - x^2$

$y - 3 = \pm\sqrt{9-x^2}$

$y = 3 \pm \sqrt{9-x^2}$

$x^2 = 9-(y-3)^2$

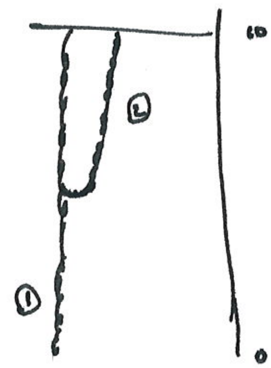
$x = \pm\sqrt{9-(y-3)^2}$

$$1000\pi \left[18y^2 - 4y^3 + \tfrac{1}{4}y^4\right]_0^6 = 1000\pi\,[648 - 864 + 324]$$

$$1000\pi(108) = 108{,}000\,\pi \qquad N\cdot m$$

7. A 10-ft chain weighs 25 lbs and hangs from a ceiling. Find the work done in lifting the lower end of the chain to the ceiling. (5 points)

$\frac{25}{10} = 2.5$ lbs/ft



$$\int_0^{10} 2.5 \cdot \tfrac{1}{2} y \, dy =$$

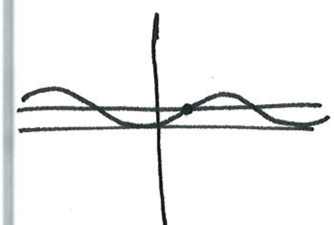
$$1.25 \int_0^{10} y\,dy = 0.625 y^2 \Big|_0^{10} =$$

$$0.625(100) = 62.5 \text{ ft.-lbs.}$$

8. Find the average value of the function $f(t) = e^{\sin t}\cos(t)$ on the interval $\left[0, \frac{\pi}{2}\right]$. Use your calculator to find c so that $f(c) = \bar{f}$. (5 points)

$$\bar{f} = \frac{2}{\pi} \int_0^{\pi/2} e^{\sin t} \cos t \, dt$$

$u = \sin t$
$du = \cos t \, dt$



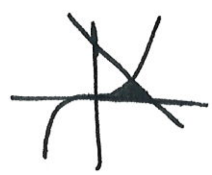
$$= \frac{2}{\pi}\left[e^{\sin t} \Big|_0^{\pi/2} = \frac{2}{\pi}[e - 1]\right]$$

$f(c) = \bar{f}$

$C \approx .08986305$

9. Set up the three integrals needed to find the centroid of the region bounded by $y = x^3$, $x + y = 2, y = 0$ with constant density. Sketch the region. You do not need to integrate. (6 points)

$$M = \int_0^1 x^3 dx + \int_1^2 -x+2 \, dx$$



$$M_y = \int_0^1 x \cdot x^3 dx + \int_1^2 x(-x+2)\,dx$$

$$M_x = \int_0^1 \tfrac{1}{2}(x^3)^2 dx + \int_1^2 \tfrac{1}{2}(-x+2)^2 dx$$

$$\bar{x} = \frac{M_y}{M}, \quad \bar{y} = \frac{M_x}{M}$$

10. Find the solution to $y' = \frac{xy\sin x}{y+1}, y(0) = 1$. (5 points)

$$\frac{y+1}{y}\,dy = x\sin x\,dx$$

u	dv
+ x	sin x
− 1	− cos x
0	− sin x

$$\int 1+\frac{1}{y}\,dy = \int x\sin x\,dx$$

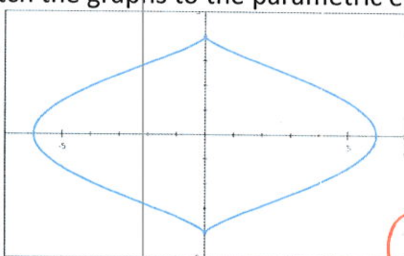
$$y+\ln y = -x\cos x+\sin x + C$$

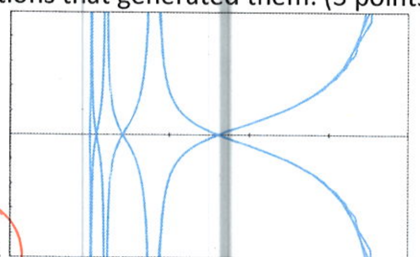
$$1+\cancel{\ln 1}^{0} = -0\cancel{\cos 0}^{0} + \cancel{\sin(0)}^{0} + C$$

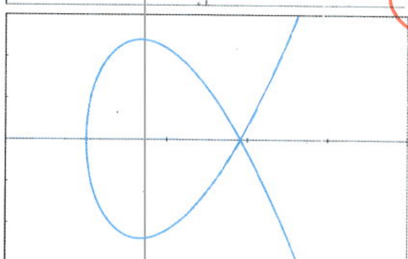
$$C = 1$$

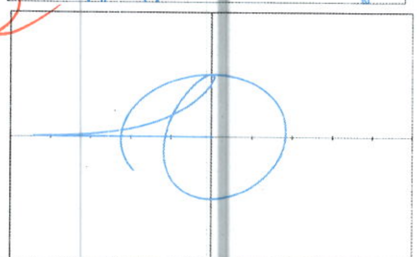
$$y+\ln y = -x\cos x+\sin x+1$$

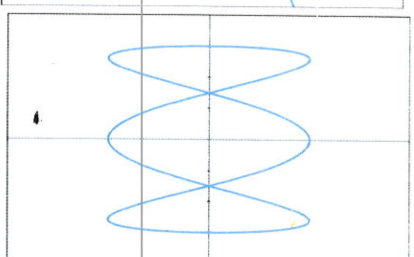
11. Match the graphs to the parametric equations that generated them. (3 points each)

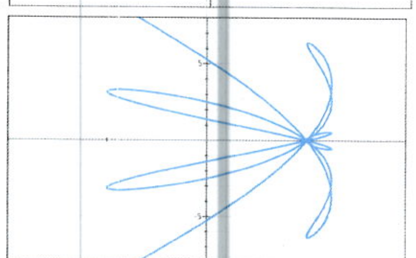
IV.

a.


c.
ii

iii

b.


d.
i

V

c.


e.
vi

i. $x = \ln t\cos t\,, y = \sin t$ D

ii. $x = \sec t\,, y = \tan(8t)$ C

iii. $x = \cosh(t)\,, y = (t^2-3)\sinh(t)$ B

iv. $x = 6\sin^3 t\,, y = 4\cos t$ A

v. $x = \sin(3t)\,, y = 3\cos t$ C

vi. $x = \sin^2 t + \cos t\,, y = t\cos t$ E

12. Find the equation of the tangent line to the graph $x = t - t^{-1}, y = 1 + t^2$ at $t = 1$. (5 points)

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$ $\frac{dx}{dt} = 1 + \frac{1}{t^2}$ $\frac{dy}{dt} = 2t$

$\frac{dy}{dx}(1) = \frac{2(1)}{1+\frac{1}{1^2}} = \frac{2}{2} = 1$

$y(1) = 2$

$x(1) = 0$

$y - 2 = 1(x - 0)$

$\boxed{y = x + 2}$

13. Set up the integral to find the length of arc of the curve $x = t^4 - 2t^3 - 2t^2, y = t^3 - 1$ on the interval $[0,2]$. You do not need to integrate. (5 points)

$\frac{dy}{dt} = 3t^2$ $\frac{dx}{dt} = 4t^3 - 6t^2 - 4t$

$$S = \int_0^2 \sqrt{(3t^2)^2 + (4t^3 - 6t^2 - 4t)^2}\, dt$$

14. Set up the integral to find the area inside $r = 3 + 2\cos\theta$ and $r = 3 + 2\sin\theta$. (5 points)

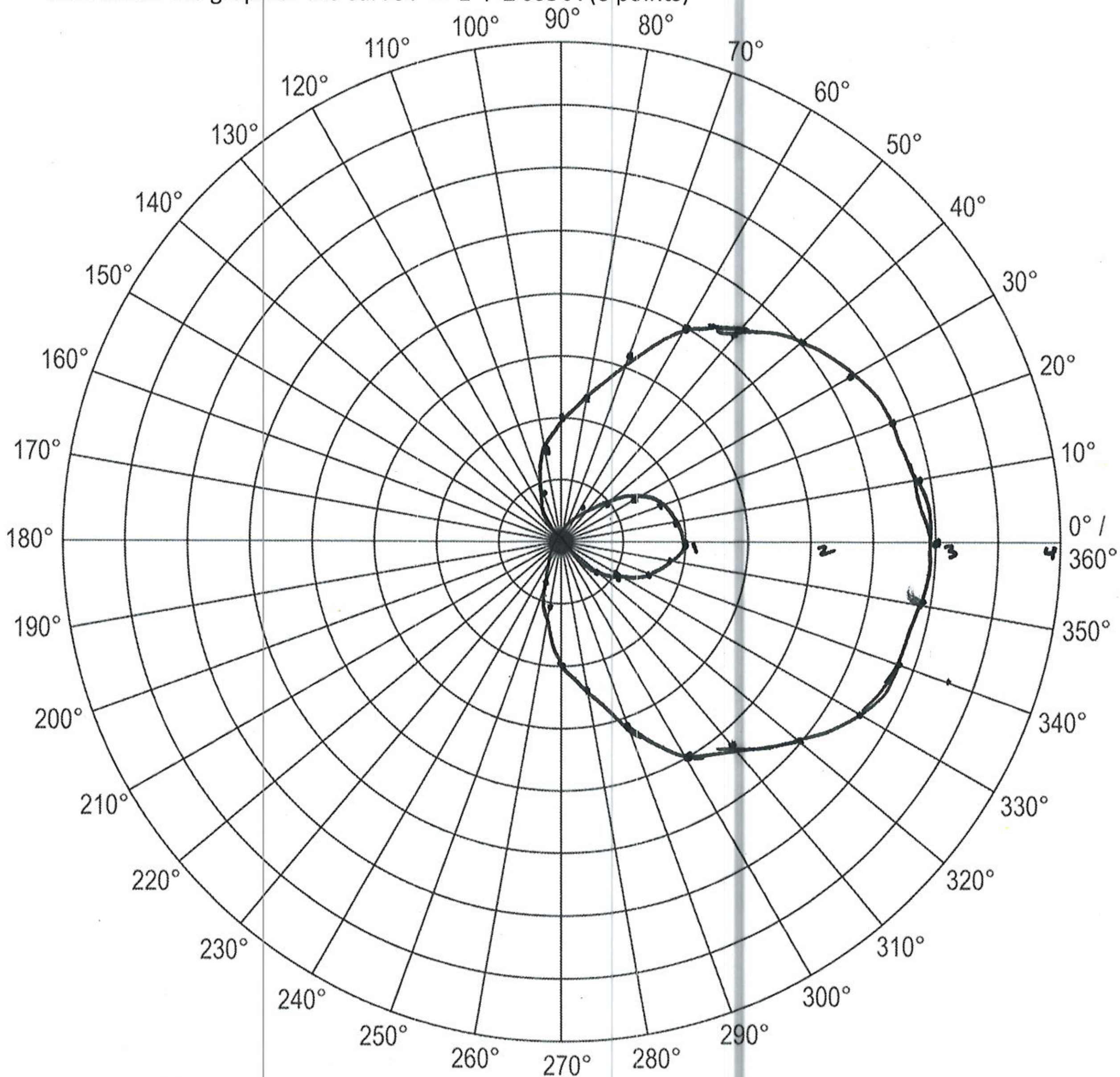
$3 + 2\cos\theta = 3 + 2\sin\theta$

$\cos\theta = \sin\theta$

$\pi/4$, $5\pi/4$, etc

$$\frac{1}{2}\int_{-3\pi/4}^{\pi/4} (3+2\sin\theta)^2\, d\theta + \frac{1}{2}\int_{\pi/4}^{5\pi/4} (3+2\cos\theta)^2\, d\theta$$

15. Sketch the graph of the curve $r = 1 + 2\cos\theta$. (6 points)



16. Write the sequence $\left\{\frac{1}{2}, -\frac{4}{3}, \frac{9}{4}, -\frac{16}{5}, \frac{25}{6}, \dots\right\}$ as a formula for a_n in terms of n. (4 points)

Start $n=1$

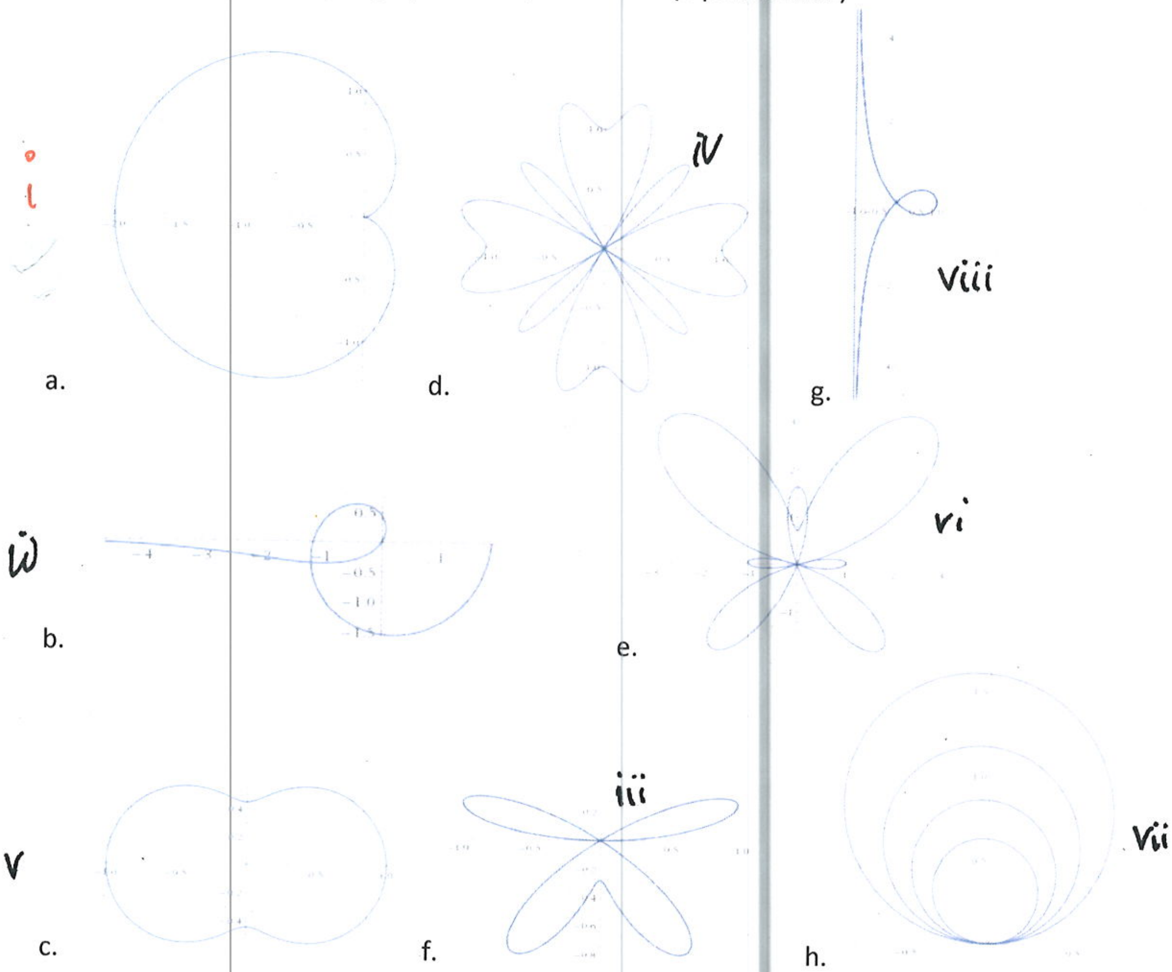
$$a_n = \frac{(-1)^{n+1} n^2}{n+1}$$

or

Start $n=0$

$$a_n = \frac{(-1)^n (n+1)^2}{n+2}$$

17. Match the equation to the graph of the polar curve. (3 points each)



i. $r = 1 - \cos\theta$ A

ii. $r = \ln\theta$ B

iii. $r = \sin(6\sin\theta)$ F

iv. $r = \sin^2(4\theta) + \cos(4\theta)$ D

v. $r = \sqrt{1 - \frac{4}{5}\sin^2\theta}$ C

vi. $r = e^{\sin\theta} - 2\cos(4\theta)$ E

vii. $r = e^{\theta/10}\sin\theta$ H

viii. $r = 2\cos\theta - \sec\theta$ G

18. Determine whether the sequence converges or diverges. If it converges, what does it converge to? (4 points each)

a. $a_n = \frac{n^3}{n+1}$

diverges

$$\lim_{n\to\infty} \frac{n^3}{n+1} = \infty$$

b. $a_n = e^{2n/(n+2)}$

$$n+2 \overline{)\,2n\,}^{\;2} \quad \frac{2n+4}{-4}$$

$$\lim_{n\to\infty} e^{\frac{2n}{n+2}} = \lim_{n\to\infty} e^{2-\frac{4}{n+2}} =$$

$$\lim_{n\to\infty} e^2 \cdot e^{-4/n+2} = e^2 \cdot e^{\lim_{n\to\infty} \frac{-4}{n+2}} =$$

$$e^2 \cdot e^0 = e^2$$

Converges

19. Find the sum of the series, if it exists. (4 points each)

a. $\sum_{n=1}^{\infty} \frac{3^{n+1}}{\pi^n}$ $\sum_{n=1}^{\infty} 3\left(\frac{3}{\pi}\right)^n$ geometric

$$\frac{3\left(\frac{3}{\pi}\right)}{1-\frac{3}{\pi}} = \frac{9/\pi}{\frac{\pi-3}{\pi}} = \frac{9}{\cancel{\pi}} \frac{\cancel{\pi}}{\pi-3} = \frac{9}{\pi-3}$$

c. $\sum_{n=1}^{\infty} (e^{1/n} - e^{1/(n+1)})$

telescoping

$$e^1 - \lim_{n \to \infty} e^{\frac{1}{n+1}} = e-1$$

b. $\sum_{n=3}^{\infty} \left(\frac{3}{n}\right)^4$

$$= \sum_{n=3}^{\infty} \frac{81}{n^4} = \sum_{n=1}^{\infty} \frac{81}{n^4} - 81 - \frac{81}{16}$$

$$= \frac{81\pi^4}{90} - 81 - \frac{81}{16} = \boxed{\frac{9\pi^4}{10} - \frac{1377}{16}}$$

20. For each series select an appropriate test to determine convergence or divergence of the series. (5 points each)

a. $\sum_{n=1}^{\infty} \frac{n^2 2^{n-1}}{(-5)^n}$

ratio or root test

Converges

d. $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$

integral test

diverges

b. $\sum_{n=1}^{\infty} \frac{\sin 2n}{1+2^n}$

(limit) Comparison test

Converges

e. $\sum_{k=2}^{\infty} \frac{k \ln k}{(k+1)^3}$

(direct) comparison test

Converges

c. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\cosh n}$

alternating series test

Converges

21. Determine the interval of convergence of the radius of convergence for the power series $\sum_{n=1}^{\infty} \frac{n(2x-1)^n}{5^n}$. Be sure to check the endpoints of the interval. (5 points)

$$\lim_{n\to\infty} \left| \frac{(n+1)(2x-1)^{n+1}}{5^{n+1}} \cdot \frac{5^n}{n(2x-1)^n} \right| = \lim_{n\to\infty} \left| \frac{2x-1}{5} \right| < 1$$

$$-1 < \frac{2x-1}{5} < 1$$

$$-\frac{5}{2} < \frac{2x-1}{2} < \frac{5}{2}$$

$$-\frac{5}{2} < x - \frac{1}{2} < \frac{5}{2}$$

$$+\frac{1}{2} \qquad +\frac{1}{2} \qquad +\frac{1}{2}$$

$$-2 < x < 3$$

$$(-2, 3)$$

$$R = 5/2$$

at $x = -2$

$$\sum \frac{n(-5)^n}{5^n} = \sum n(-1)^n \text{ diverges}$$

$$\sum \frac{n\,5^n}{5^n} = \sum n \text{ diverges}$$

22. Find a power series for the function $f(x) = \frac{2}{3-x}$. (5 points)

$$= \frac{2/3}{1 - \frac{x}{3}}$$

$a = 2/3, \; r = \frac{x}{3}$

$$f(x) = \sum_{n=0}^{\infty} \left(\frac{2}{3}\right)\left(\frac{x}{3}\right)^n$$

23. Use a power series to evaluate $\int \frac{t}{1+t^5}\,dt$. (5 points)

$r = -t^5, \; a = t$

$$\int \sum_{n=0}^{\infty} t(-t^5)^n \, dt = \sum_{n=0}^{\infty} (-1)^n \int t^{5n+1} \, dt =$$

$$\sum_{n=0}^{\infty} \frac{(-1)^n t^{5n+2}}{5n+2} + C$$

24. Rewrite $x^5 + 3x^4 - 2x$ as a Taylor series centered at $c = 1$. (5 points)

n	$n!$	$f^{(n)}(x)$	$f^{(n)}(c)$	$(x-c)^n$	$\frac{f^{(n)}(c)}{n!}(x-c)^n$
0	1	x^5+3x^4-2x	2	1	2
1	1	$5x^4+12x^3-2$	15	$(x-1)$	$15(x-1)$
2	2	$20x^3+36x^2$	56	$(x-1)^2$	$28(x-1)^2$
3	6	$60x^2+72x$	132	$(x-1)^3$	$22(x-1)^3$
4	24	$120x+72$	192	$(x-1)^4$	$8(x-1)^4$
5	120	120	120	$(x-1)^5$	$(x-1)^5$
6	720	0	—	$(x-1)^6$	—

$P_n(x) = (x-1)^5 + 8(x-1)^4 + 22(x-1)^3 + 28(x-1)^2 + 15(x-1) + 2$

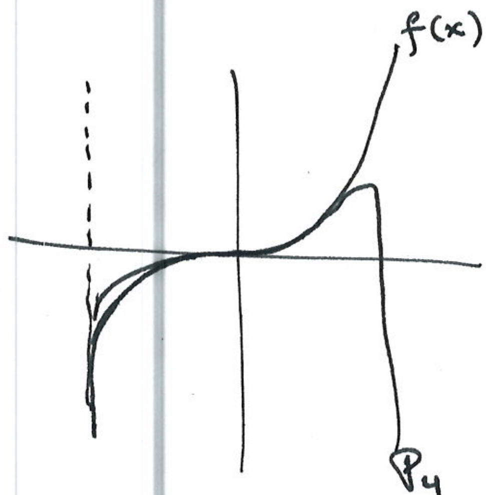
25. Find a Maclaurin series for $f(x) = x^2 \ln(1 + x^3)$. Use the table of Maclaurin series included at the end of the exam. Graph the first 4 (non-zero) terms on the same graph as the original function. (5 points)

$$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n}$$

$$\ln(1+x^3) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^{3n}}{n}$$

$$x^2 \ln(1+x^3) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^{3n+2}}{n}$$

$$P_4(x) = x^5 - \frac{x^8}{2} + \frac{x^{11}}{3} - \frac{x^{14}}{4}$$

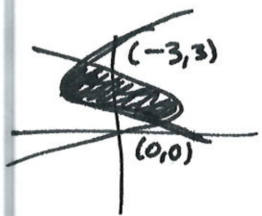


26. Find the area of the region bounded by $x = y^2 - 4y$ and $x = 2y - y^2$. Sketch the region. (6 points)

$y^2 - 4y = 2y - y^2$

$\frac{2y^2 = 6y}{y}$ $\quad y = 0$

$y = 3$



$\int_0^3 2y - y^2 - (y^2 - 4y)dy = \int_0^3 6y - 2y^2\, dy$

$3y^2 - \frac{2}{3}y^3 \Big|_0^3 = 27 - \frac{2}{3}(27) =$

$27 - 18 = 9$

27. Which method of integration should be used to evaluate each integral? You do not need to integrate. If you are using a substitution method, state the substitution used. (3 points each)

a. $\int \cos x \ln(\sin x)\, dx$

$u = \sin x$
$du = \cos x$

by parts

d. $\int \cos^2 \theta \sin^2 \theta \, d\theta$

identities /

substitution

b. $\int x^5 e^{-x^3} dx$

by parts

e. $\int \sqrt{x^2 + 2x}dx$

trig substitution

c. $\int \frac{t}{t^4+2} dt$

substitution $t^2 = u$

arctan rule

f. $\int \frac{x^3 - 4x - 10}{x^2 - x - 6} dx$

partial fractions after long division

28. Integrate. (5 points each)

a. $\int \sin 8x \cos 5x \, dx$

$= \frac{1}{2}\int \sin 13x + \sin 3x\, dx = -\frac{1}{26}\cos 13x - \frac{1}{6}\cos 3x + C$

b. $\int \frac{dx}{\cos x-1}$ $\frac{\cos x+1}{\cos x+1} = \int \frac{\cos x+1}{\cos^2 x-1}\,dx = \int \frac{\cos x+1}{\sin^2 x}\,dx$

$= \int \csc x \cot x + \csc^2 x \, dx$

$= -\csc x - \cot x + C$

29. Apply partial fractions to the expression $\frac{1}{(x^2-4)(x^2-x-6)(x^2+1)x^3}$. You do not need to integrate or solve for the coefficients. (5 points)

$\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{x-2} + \frac{E}{(x+2)} + \frac{F}{(x+2)^2} + \frac{G}{(x-3)} + \frac{Hx+I}{x^2+1}$

30. Use Simpson's Rule to estimate $\int_0^2 \frac{1}{1+x^5}\,dx$ with $n = 4$. (6 points)

$h = \Delta x = \frac{1}{2}$

$f(x) = \frac{1}{6}\left[\frac{1}{1+0^5} + \frac{4}{1+(1/2)^5} + \frac{2}{1+1^5} + \frac{4}{1+(3/2)^5} + \frac{1}{1+2^5} \right]$

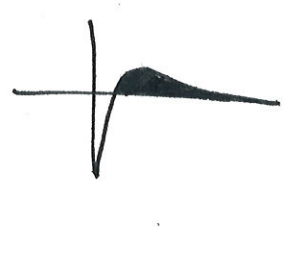
$\approx 1.0624 \ldots.$

31. Determine if the improper integral $\int_1^\infty \frac{\ln x}{x}\,dx$ converge or diverge. Sketch the graph of the region to find all points of discontinuity insider the interval. If it converges, evaluate it. (5 points)

$u = \ln x \quad du = \frac{1}{x}\,dx$

$\int u\,du = \frac{1}{2}u^2$

$\lim_{b \to \infty} \frac{1}{2}\ln^2 b - \frac{1}{2}\ln(1) = \infty$



diverges

$$\arcsin x = x + \frac{x^3}{2\cdot 3} + 1\cdot\frac{3x^5}{2\cdot 4\cdot 5\cdot 7} + \cdots + \frac{(2n)!\,x^{2n+1}}{(2^n n!)^2(2n+1)} \qquad -1 < x < 1$$

$$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \cdots = \sum_{n=0}^{\infty} \frac{1}{(2n)!} x^{2n} \qquad (-\infty, \infty)$$

$$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \cdots = \sum_{n=0}^{\infty} \frac{1}{(2n+1)!} x^{2n+1} \qquad (-\infty, \infty)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$\cos^2 t = \frac{1}{2}(1 + \cos 2t)$$

$$\sin 2t = 2\sin t\cos t\,, \cos 2t = \cos^2 t - \sin^2 t$$

$$\cos a\cos b = \frac{1}{2}[\cos(a+b) + \cos(a-b)]$$
$$\sin a\sin b = \frac{1}{2}[\cos(a-b) - \cos(a+b)]$$
$$\sin a\cos b = \frac{1}{2}[\sin(a+b) + \sin(a-b)]$$

Trapezoidal Rule Error: $|E| \leq \frac{(b-a)^3}{12n^2}[\max|f''(x)|]$

Simpson's Rule Error: $|E| \leq \frac{(b-a)^5}{180n^4}[\max|f^{IV}(x)|]$

Simpson's Rule: $f(x) \approx \frac{b-a}{3n}[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 4f(x_{n-1}) + f(x_n)]$

Trapezoidal Rule: $f(x) \approx \frac{b-a}{2n}[f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)]$

$$M = \rho\int_a^b f(x) - g(x)dx$$
$$M_x = \rho\int_a^b \frac{1}{2}\left(f^2(x) - g^2(x)\right)dx$$
$$M_y = \rho\int_a^b x\left(f(x) - g(x)\right)dx$$
$$\bar{x} = \frac{M_y}{M}, \bar{y} = \frac{M_x}{M}$$

Some useful formulas:

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left[\frac{dy}{dx}\right]}{\frac{dx}{dt}}$$

$$\frac{dy}{dx} = \frac{r(\theta)cos\theta + r'(\theta)\sin\theta}{-r(\theta)\sin\theta + r'(\theta)\cos\theta}$$

$$s = \int_{\alpha}^{\beta} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}, \qquad \sum_{n=1}^{\infty} 1/n^4 = \frac{\pi^4}{90}$$

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(c)(x-c)^n}{n!}$$

$$|R_n(x)| \leq \frac{\max|f^{n+1}(z)|}{(n+1)!} x^{n+1}$$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \dots \qquad R = 1$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots \qquad R = \infty$$

$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots \qquad R = \infty$$

$$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots \qquad R = \infty$$

$$\tan^{-1} x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots \qquad R = 1$$

$$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots \qquad R = 1$$

$$(1+x)^k = \sum_{n=0}^{\infty} \binom{k}{n} x^n = 1 + kx + \frac{k(k-1)}{2!} x^2 + \frac{k(k-1)(k-2)}{3!} x^3 + \dots \qquad R = 1$$