MAT 192, Exam #1, Part I, Fall 2016 Name KEY

Instructions: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate. This portion of the exam is completed with the table of integrals provided. When completed, submit this portion along with the table of integrals and pick up the remainder of the exam.

1. For each integral below, use the table of integrals to integrate. In each case, list any substitutions made and the formula number you used.

a. $\int \frac{\cos\theta}{3+2\sin\theta+\sin^2\theta}\,d\theta$

$u = \sin\theta$, $du = \cos\theta$

$\int \frac{du}{3+2u+u^2}$

$a=1,\ b=2,\ c=3$

$4 < 4(1)(3) = 12$

$b^2 < 4ac$

formula 2.23

$\int \frac{1}{ax^2+bx+c}\,dx = \frac{2}{\sqrt{4ac-b^2}}\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)+C$

$= \frac{2}{\sqrt{8}}\arctan\left(\frac{2u+2}{\sqrt{8}}\right)+C = \boxed{\frac{2}{\sqrt{8}}\arctan\left(\frac{2\sin\theta+2}{\sqrt{8}}\right)+C}$

b. $\int x\,\mathrm{arccsc}(x^2+1)\,dx$ $u = x^2+1$ $du = 2x\,dx \Rightarrow \frac{1}{2}du = x\,dx$

$\frac{1}{2}\int \mathrm{arccsc}\,u\,du$

formula 5.14

$\int \mathrm{arccsc}\,x\,dx = x\,\mathrm{arccsc}\,x + \ln\left|x+\sqrt{x^2-1}\right| + C$

$= \frac{1}{2}\left[u\,\mathrm{arccsc}\,u + \ln\left|u+\sqrt{u^2-1}\right|\right]+C = \boxed{\frac{1}{2}\left[(x^2+1)\,\mathrm{arccsc}(x^2+1) + \ln\left|(x^2+1)+\sqrt{(x^2+1)^2-1}\right|\right] + C}$

c. $\int \frac{x^2}{(2x-7)^2}\,dx$ $a=2,\ b=-7$

formula 2.9

$\int \frac{x^2}{(ax+b)^2}\,dx = \frac{x}{a^2} - \frac{b^2}{a^3(ax+b)} - \frac{2b}{a^3}\ln|ax+b| + C$

$\boxed{\frac{x}{4} - \frac{49}{8(2x-7)} + \frac{14}{8}\ln|2x-7| + C}$

d. $\int e^x \sqrt{\frac{e^x-5}{e^x+5}}\,dx$

$u = e^x$
$du = e^x dx$

$\int \sqrt{\frac{u-5}{u+5}}\,du$

formula 3.16

$\int \sqrt{\frac{x-a}{x+a}}\,dx = \sqrt{x^2-a^2} - a\ln\left|\sqrt{x^2-a^2}+x\right| + C$ $\quad a=5$

$= \sqrt{u^2-25} - 5\ln\left|\sqrt{u^2-25}+u\right| + C$

$= \boxed{\sqrt{e^{2x}-25} - 5\ln\left|\sqrt{e^{2x}-25}+e^x\right| + C}$

e. $\int \frac{\cos^3(\ln x)}{x}\,dx$

$u = \ln x$
$du = \frac{1}{x}\,dx$

$\int \cos^3 u\,du$

formula 4.11

$\int \cos^3(ax)\,dx = \frac{1}{3a}\sin(ax)\cos^2(ax) + \frac{2}{3a}\sin(ax) + C$ $\quad a=1$

$\frac{1}{3}\sin u\cos^2 u + \frac{2}{3}\sin u + C$

$= \boxed{\frac{1}{3}\sin(\ln x)\cos^2(\ln x) + \frac{2}{3}\sin(\ln x) + C}$

Table of Integrals

Basic Rules

(1.1) $$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

(1.2) $$\int \frac{1}{x} dx = \ln|x| + C$$

(1.3) $$\int e^x dx = e^x + C$$

(1.4) $$\int a^x dx = \frac{a^x}{\ln a} + C$$

(1.5) $$\int \sin x dx = -\cos x + C$$

(1.6) $$\int \cos x dx = \sin x + C$$

(1.7) $$\int \tan x dx = -\ln|\cos x| + C = \ln|\sec x| + C$$

(1.8) $$\int \cot x dx = \ln|\sin x| + C$$

(1.9) $$\int \sec^2 x dx = \tan x + C$$

(1.10) $$\int \sec x dx = \ln|\sec x + \tan x| + C = \ln\left|\frac{1+\sin x}{\cos x}\right| + C$$

(1.11) $$\int \csc^2 x dx = -\cot x + C$$

(1.12) $$\int \csc x dx = -\ln|\csc x + \cot x| + C = \ln\left|\frac{\sin x}{1+\cos x}\right| + C$$

(1.13) $$\int \sec x \tan x dx = \sec x + C$$

(1.14) $$\int \csc x \cot x dx = -\csc x + C$$

(1.15) $$\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin x + C$$

(1.16) $$\int \frac{1}{x\sqrt{x^2-1}} dx = \operatorname{arcsec}|x| + C$$

(1.17) $$\int \frac{1}{1+x^2} dx = \arctan x + C$$

(1.18) $$\int \sinh x dx = \cosh x + C$$

(1.19) $$\int \cosh x dx = \sinh x + C$$

(1.20) $$\int \tanh x dx = \ln|\cosh x| + C$$

(1.21) $$\int \operatorname{sech}^2 x dx = \tanh x + C$$

(1.22) $$\int \coth x dx = \ln|\sinh x| + C$$

(1.23) $$\int \operatorname{sech} x \tanh x dx = -\operatorname{sech} x + C$$

(1.24) $$\int \operatorname{csch} x \coth x dx = -\operatorname{csch} x + C$$

(1.25) $$\int \operatorname{csch}^2 x dx = -\coth x + C$$

(1.26) $$\int \frac{1}{\sqrt{x^2+1}} dx = \sinh^{-1} x + C$$

(1.27) $$\int \frac{1}{\sqrt{x^2-1}} dx = \cosh^{-1} x + C$$

(1.28) $$\int \frac{1}{1-x^2} dx = \tanh^{-1} x + C$$

(1.29) $$\int \frac{1}{x\sqrt{1-x^2}} dx = -\operatorname{sech}^{-1} x + C$$

(1.30) $$\int \frac{1}{x\sqrt{1+x^2}} dx = -\operatorname{csch}^{-1} x + C$$

Rational and Polynomial Functions

(2.1) $$\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C, n \neq -1$$

(2.2) $$\int x(ax+b)^n dx = \frac{x(ax+b)^{n+1}}{a(n+1)} - \frac{(ax+b)^{n+1}}{a(n+1)(n+2)} + C, n \neq -1, -2$$

(2.3) $$\int \frac{1}{ax+b} dx = \frac{1}{a} \ln|ax+b| + C$$

(2.4) $$\int \frac{1}{(x+a)^2} dx = -\frac{1}{x+a} + C$$

(2.5) $$\int \frac{x}{x+a} dx = \frac{a}{x+a} + \ln|x+a| + C$$

(2.6) $$\int \frac{x}{ax+b} dx = \frac{x}{a} - \frac{b}{a^2} \ln|ax+b| + C$$

(2.7) $$\int \frac{x^2}{ax+b} dx = \frac{x^2}{2a} - \frac{bx}{a^2} + \frac{b^2}{a^3} \ln|ax+b| + C$$

(2.8) $$\int \frac{x}{(ax+b)^2} dx = \frac{1}{a^2}\left(\ln|ax+b| + \frac{b}{ax+b} \right) + C$$

(2.9) $$\int \frac{x^2}{(ax+b)^2} dx = \frac{x}{a^2} - \frac{b^2}{a^3(ax+b)} - \frac{2b}{a^3} \ln|ax+b| + C$$

(2.10) $$\int \frac{x}{(ax+b)^n} dx = \frac{1}{a^2}\left[\frac{b}{(n-1)(ax+b)^{n-1}} - \frac{1}{(n-2)(ax+b)^{n-2}} \right] + C, n \neq 1, 2$$

(2.11) $$\int \frac{x^2}{(ax+b)^3}dx = \frac{1}{a^3}\left[\frac{2b}{ax+b} - \frac{b^2}{2(ax+b)^2} + \ln|ax+b|\right] + C$$

(2.12) $$\int \frac{x^2}{(ax+b)^n}dx = \frac{1}{a^3}\left[\frac{2b}{(n-2)(ax+b)^{n-2}} - \frac{1}{(n-3)(ax+b)^{n-3}} - \frac{b^2}{(n-1)(ax+b)^{n-1}}\right] + C, n \neq 1,2,3$$

(2.13) $$\int \frac{1}{a^2+x^2}dx = \frac{1}{a}\arctan\left(\frac{x}{a}\right) + C$$

(2.14) $$\int \frac{x}{a^2+x^2}dx = \frac{1}{2}\ln\left(a^2+x^2\right) + C$$

(2.15) $$\int \frac{x^2}{a^2+x^2}dx = x - a\arctan\left(\frac{x}{a}\right) + C$$

(2.16) $$\int \frac{x^3}{a^2+x^2}dx = \frac{1}{2}x^2 - \frac{1}{2}a^2\ln\left(a^2+x^2\right) + C$$

(2.17) $$\int \frac{1}{x(ax+b)}dx = \frac{1}{b}\ln\left|\frac{x}{ax+b}\right| + C$$

(2.18) $$\int \frac{1}{x^2(ax+b)}dx = \frac{a}{b^2}\ln\left|\frac{ax+b}{x}\right| - \frac{1}{bx} + C$$

(2.19) $$\int \frac{1}{x(ax+b)^2}dx = \frac{1}{b(ax+b)} + \frac{1}{b^2}\ln\left|\frac{x}{ax+b}\right| + C$$

(2.20) $$\int \frac{1}{x^2(ax+b)^2}dx = \frac{2a}{b^3}\ln\left|\frac{ax+b}{x}\right| - \frac{2ax+b}{b^2x(ax+b)} + C$$

(2.21) $$\int \frac{1}{(ax+b)(cx+d)}dx = \frac{1}{ad-bc}\ln\left|\frac{ax+b}{cx+d}\right| + C$$

(2.22) $$\int \frac{x}{(ax+b)(cx+d)}dx = \frac{1}{ad-bc}\left[\frac{d}{c}\ln|cx+d| - \frac{b}{a}\ln|ax+b|\right] + C$$

(2.23) $$\int \frac{1}{ax^2+bx+c}dx = \begin{cases} \frac{2}{\sqrt{4ac-b^2}}\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right) + C, b^2 < 4ac \\ \frac{1}{\sqrt{b^2-4ac}}\ln\left|\frac{2cu+b-\sqrt{b^2-4ac}}{2cu+b+\sqrt{b^2-4ac}}\right| + C, b^2 > 4ac \end{cases}, b^2 \neq 4ac$$

(2.24)

$$\int \frac{x}{ax^2+bx+c}dx = \frac{1}{2a}\ln|ax^2+bx+c| - \begin{cases} \frac{b}{a\sqrt{4ac-b^2}}\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right) + C, b^2 < 4ac \\ \frac{1}{\sqrt{b^2-4ac}}\ln\left|\frac{2cu+b-\sqrt{b^2-4ac}}{2cu+b+\sqrt{b^2-4ac}}\right| + C, b^2 > 4ac \end{cases}, b^2 \neq 4ac$$

(2.25) $$\int \frac{1}{a^2-x^2}dx = \frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right| + C$$

(2.26) $$\int \frac{1}{x^2-a^2}dx = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + C$$

(2.27) $$\int \frac{1}{(a^2 \pm x^2)^n}dx = \frac{1}{2a^2(n-1)}\left[\frac{x}{(a^2 \pm x^2)^{n-1}} + (2n-3)\int \frac{1}{(a^2 \pm x^2)^{n-1}}dx\right] + C, n \neq -1$$

(2.28) $$\int \frac{x^n}{x^2 \pm 1}dx = \frac{1}{n-1}x^{n-1} \mp \int \frac{x^{n-2}}{x^2 \pm 1}dx + C, n \neq 0,1$$

(2.29) $$\int \frac{dx}{x\left(x^2 \pm a^2\right)} = \pm\frac{1}{2a^2}\ln\left|\frac{x^2}{a^2 \pm x^2}\right| + C$$

Radical Functions

(3.1) $$\int \sqrt{ax+b}dx = \frac{2}{3a}(ax+b)^{3/2} + C$$

(3.2) $$\int \frac{1}{\sqrt{ax+b}}dx = \frac{2}{a}\sqrt{ax+b} + C$$

(3.3) $$\int (ax+b)^{(2n+1)/2}dx = \frac{2}{a(2n+3)}(ax+b)^{(2n+3)/2} + C$$

(3.4) $$\int x\sqrt{ax+b}dx = \frac{2x}{5a}(ax+b)^{3/2} - \frac{4b}{15a^2}(ax+b)^{5/2} + C$$

(3.5) $$\int x^2\sqrt{ax+b}dx = \frac{2x^2}{3a}(ax+b)^{3/2} + \frac{8x}{15a^2}(ax+b)^{5/2} + \frac{16}{105a^3}(ax+b)^{7/2} + C$$

(3.6) $$\int x^m(ax+b)^{(2n+1)/2}dx = \frac{2}{a(2n+3)}\left[x^m(ax+b)^{(2n+3)/2} - m\int x^{m-1}(ax+b)^{(2n+3)/2}\right] + C$$

(3.7) $$\int \frac{x}{\sqrt{ax+b}}dx = \frac{2x}{a}\sqrt{ax+b} - \frac{4b}{3a^2}(ax+b)^{3/2} + C$$

(3.8) $$\int \frac{x^2}{\sqrt{ax+b}}dx = \frac{2}{15a^3}\left(2a^2x^2 - 4abx + 18b^2\right)\sqrt{ax+b}dx$$

(3.9) $$\int \frac{x^n}{\sqrt{ax+b}}dx = \frac{2}{(2n+1)a}\left[x^n\sqrt{ax+b} - nb\int \frac{x^{n-1}}{\sqrt{ax+b}}dx\right] + C$$

(3.10) $$\int \frac{1}{x\sqrt{ax+b}}dx = \begin{cases} \frac{1}{\sqrt{b}}\ln\left|\frac{\sqrt{ax+b}-\sqrt{b}}{\sqrt{ax+b}-\sqrt{b}}\right| + C, a > 0 \\ \frac{2}{\sqrt{-b}}\arctan\sqrt{\frac{ax+b}{-b}} + C, a < 0 \end{cases}$$

(3.11) $$\int \frac{1}{x^n\sqrt{ax+b}}dx = \frac{1}{a(1-n)}\left[\frac{\sqrt{ax+b}}{x^{n-1}} + \frac{2(n-3)a}{2}\int \frac{1}{x^{n-1}\sqrt{ax+b}}dx\right] n \neq 1$$

(3.12) $$\int \frac{\sqrt{ax+b}}{x}dx = 2\sqrt{ax+b} + b\begin{cases} \frac{1}{\sqrt{b}}\ln\left|\frac{\sqrt{ax+b}-\sqrt{b}}{\sqrt{ax+b}-\sqrt{b}}\right| + C, a>0 \\ \frac{2}{\sqrt{-b}}\arctan\sqrt{\frac{ax+b}{-b}} + C, a<0 \end{cases}$$

(3.13) $$\int \frac{\sqrt{ax+b}}{x^n}dx = \frac{1}{b(1-n)}\left[\frac{(ax+b)^{3/2}}{x^{n-1}} + \frac{(2n-5)a}{2}\int \frac{1}{x^{n-1}\sqrt{ax+b}}dx\right] + C, n \neq 1$$

(3.14) $$\int \sqrt{\frac{x}{a-x}}dx = \frac{a}{2}\arcsin\left(\frac{a-2x}{a}\right) + \sqrt{x(a-x)} + C$$

(3.15) $$\int \sqrt{\frac{x}{x+a}}dx = \sqrt{x(x+a)} - \frac{a}{2}\ln\left|\sqrt{x(a+x)} + \left(x+\frac{a}{2}\right)\right| + C$$

(3.16) $$\int \sqrt{\frac{x-a}{x+a}}dx = \sqrt{x^2-a^2} - a\ln\left|\sqrt{x^2-a^2}+x\right| + C$$

(3.17) $$\int \sqrt{\frac{x+a}{x+b}}dx = \sqrt{(x+a)(x+b)} + (a-b)\ln\left|\sqrt{x+a}+\sqrt{x+b}\right| + C$$

(3.18) $$\int \frac{1}{\sqrt{(x+a)(x+b)}}dx = \ln\left|\frac{a+b}{2} + x + \sqrt{(x+a)(x+b)}\right| + C$$

(3.19) $$\int \sqrt{x(ax+b)}dx = \frac{x\sqrt{x(ax+b)}}{2} + \frac{b}{4a}\sqrt{x(ax+b)} - \frac{b^2}{4a^{3/2}}\ln\left|\sqrt{ax}+\sqrt{ax+b}\right| + C$$

(3.20) $$\int \sqrt{ax^2+bx+c}dx = \frac{2ax+b}{4a}\sqrt{ax^2+bx+c} - \frac{b^2-4ac}{8a^{3/2}}\ln\left|2ax+b+2\sqrt{a(ax^2+bx+c)}\right| + C$$

(3.21)

$$\int x\sqrt{ax^2+bx+c}dx = \frac{\sqrt{ax^2+bx+c}}{24a^2} - \frac{4ax^2+bx+4c}{24a^{3/2}} + \frac{b(b^2-4ac)}{16a^{3/2}}\ln\left|b+2ax+2\sqrt{a(ax^2+bx+c)}\right| + C$$

(3.22) $$\int \frac{1}{\sqrt{ax^2+bx+c}}dx = \frac{1}{\sqrt{a}}\ln\left|2ax+b+2\sqrt{a(ax^2+bx+c)}\right| + C$$

(3.23) $$\int \frac{x}{\sqrt{ax^2+bx+c}}dx = \frac{1}{a}\sqrt{ax^2+bx+c} - \frac{b}{2a\sqrt{a}}\ln\left|2ax+b+2\sqrt{a(ax^2+bx+c)}\right| + C$$

(3.24) $$\int \sqrt{2ax-x^2}dx = \frac{x-a}{2}\sqrt{2ax-x^2} + \frac{a^2}{2}\arcsin\left(\frac{x-a}{2}\right) + C, a>0$$

(3.25) $$\int \frac{dx}{\sqrt{2ax-x^2}} = \arcsin\left(\frac{x-a}{a}\right) + C, a>0$$

(3.26) $$\int \sqrt{x^2 \pm a^2}\,dx = \frac{1}{2}x\sqrt{x^2 \pm a^2} \pm \frac{1}{2}a^2 \ln\left|x + \sqrt{x^2 \pm a^2}\right| + C$$

(3.27) $$\int \sqrt{a^2 - x^2}\,dx = \frac{1}{2}x\sqrt{a^2 - x^2} + \frac{a^2}{2}\arcsin\left(\frac{x}{a}\right) + C$$

(3.28) $$\int x\sqrt{x^2 \pm a^2}\,dx = \frac{1}{3}\left(x^2 \pm a^2\right)^{3/2} + C$$

(3.29) $$\int x^2\sqrt{x^2 \pm a^2}\,dx = \frac{x}{8}\left(2x^2 \pm a^2\right)\sqrt{x^2 \pm a^2} - \frac{a^4}{8}\ln\left|x + \sqrt{x^2 \pm a^2}\right| + C$$

(3.30) $$\int x^2\sqrt{a^2 - x^2}\,dx = \frac{x}{8}(2x^2 - a^2)\sqrt{a^2 - x^2} + \frac{a^4}{8}\arcsin\left(\frac{x}{a}\right) + C$$

(3.31) $$\int \frac{\sqrt{a^2 \pm x^2}}{x}dx = \sqrt{a^2 \pm x^2} - a\ln\left|\frac{a + \sqrt{a^2 \pm x^2}}{x}\right| + C, a > 0$$

(3.32) $$\int \frac{\sqrt{x^2 - a^2}}{x}dx = \sqrt{x^2 - a^2} - a\,\mathrm{arcsec}\left|\frac{x}{a}\right| + C$$

(3.33) $$\int \frac{\sqrt{x^2 \pm a^2}}{x^2}dx = -\frac{\sqrt{x^2 \pm a^2}}{x} + \ln\left|x + \sqrt{x^2 \pm a^2}\right| + C$$

(3.34) $$\int \frac{\sqrt{a^2 - x^2}}{x^2}dx = -\frac{\sqrt{a^2 - x^2}}{x} - \arcsin\left(\frac{x}{a}\right) + C$$

(3.35) $$\int \frac{1}{\sqrt{x^2 \pm a^2}}\,dx = \ln\left|x + \sqrt{x^2 \pm a^2}\right| + C$$

(3.36) $$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \arcsin\left(\frac{x}{a}\right) + C$$

(3.37) $$\int \frac{x}{\sqrt{x^2 \pm a^2}}\,dx = \sqrt{x^2 \pm a^2} + C$$

(3.38) $$\int \frac{x}{\sqrt{a^2 - x^2}}dx = -\sqrt{a^2 - x^2} + C$$

(3.39) $$\int \frac{x^2}{\sqrt{x^2 \pm a^2}}\,dx = \frac{1}{2}x\sqrt{x^2 \pm a^2} \mp \frac{a^2}{2}\ln\left|x + \sqrt{x^2 \pm a^2}\right| + C$$

(3.40) $$\int \frac{x^2}{\sqrt{a^2 - x^2}}\,dx = -\frac{x}{2}\sqrt{a^2 - x^2} - \frac{a^2}{2}\arcsin\left(\frac{x}{a}\right) + C$$

(3.41) $$\int \frac{1}{x\sqrt{x^2 + a^2}}\,dx = \frac{1}{a}\ln\left|\frac{\sqrt{x^2 + a^2} - a}{x}\right| + C$$

(3.42) $$\int \frac{1}{x\sqrt{x^2 - a^2}}\,dx = \frac{1}{a}\,\mathrm{arcsec}\left|\frac{x}{a}\right| + C$$

(3.43) $$\int \frac{1}{x\sqrt{a^2 \pm x^2}}\,dx = -\frac{1}{a}\ln\left|\frac{a + \sqrt{a^2 \pm x^2}}{|x|}\right| + C$$

(3.44) $$\int \frac{1}{x^2\sqrt{a^2-x^2}}dx = -\frac{\sqrt{a^2-x^2}}{a^2x}+C$$

(3.45) $$\int \frac{1}{x^2\sqrt{x^2\pm a^2}}dx = \mp\frac{\sqrt{x^2\pm a^2}}{a^2x}+C$$

(3.46) $$\int (x^2\pm a^2)^{3/2}dx = \frac{x}{8}(2x^2\pm 5a^2)\sqrt{x^2\pm a^2}+\frac{3a^4}{8}\ln\left|x+\sqrt{x^2\pm a^2}\right|+C$$

(3.47) $$\int \frac{1}{(a^2\pm x^2)^{3/2}}dx = \frac{\pm x}{a^2\sqrt{a^2\pm x^2}}+C$$

(3.48) $$\int \frac{x^2}{(x^2\pm a^2)^{3/2}}dx = \frac{-x}{\sqrt{x^2\pm a^2}}+\ln\left|x+\sqrt{x^2\pm a^2}\right|+C$$

(3.49) $$\int x^n(x^2\pm 1)^{(2k+1)/2}dx = \frac{1}{2k+3}x^{n-1}(x^2\pm 1)^{(2k+3)/2}-\frac{n-1}{2k+3}\int x^{n-2}\left(x^2\pm 1\right)^{(2k+3)/2}+C, n\neq 0$$

(3.50) $$\int x^n(1-x^2)^{(2k+1)/2}dx = -\frac{1}{2k+3}x^{n-1}(1-x^2)^{(2k+3)/2}+\frac{n-1}{2k+3}\int x^{n-2}(1-x^2)^{(2k+3)/2}dx+C, n\neq 0$$

Trigonometric Functions

(4.1) $$\int \sin(ax)dx = -\frac{1}{a}\cos(ax)+C$$

(4.2) $$\int \sin^2 axdx = \frac{1}{2}x-\frac{1}{4a}\sin(2ax)+C$$

(4.3) $$\int \sin^3 axdx = -\frac{1}{3a}\sin^2(ax)\cos(ax)-\frac{2}{3a}\cos(ax)+C$$

(4.4) $$\int \sin^4 axdx = -\frac{1}{4a}\sin^3(ax)\cos(ax)-\frac{3}{8a}\sin(ax)\cos(ax)+\frac{3}{8}x+C$$

(4.5) $$\int \sin^n xdx = \frac{1}{n}\sin^{n-1}x\cos x+\frac{n-1}{n}\int \sin^{n-2}xdx$$

(4.6) $$\int x\sin(ax)dx = -\frac{1}{a}x\cos(ax)+\frac{1}{a^2}\sin(ax)+C$$

(4.7) $$\int x^2\sin(ax)dx = \frac{2}{a^3}\cos(ax)+\frac{2}{a^2}\sin(ax)-\frac{1}{a}x^2\cos(ax)+C$$

(4.8) $$\int x^n\sin xdx = -x^n\cos x+n\int x^{n-1}\cos xdx$$

(4.9) $$\int \cos axdx = \frac{1}{a}\sin ax+C$$

(4.10) $$\int \cos^2 axdx = \frac{1}{2}x+\frac{1}{4a}\sin(2ax)+C$$

(4.11) $$\int \cos^3(ax)dx = \frac{1}{3a}\sin(ax)\cos^2(ax)+\frac{2}{3a}\sin(ax)+C$$

(4.12) $$\int \cos^4(ax)dx = \frac{1}{4a}\sin(ax)\cos^3(ax) + \frac{3}{8a}\sin(ax)\cos(ax) + \frac{3}{8}x + C$$

(4.13) $$\int \cos^n x dx = \frac{1}{n}\cos^{n-1} x \sin x + \frac{n-1}{n}\int \cos^{n-2} x dx$$

(4.14) $$\int x\cos(ax)dx = \frac{1}{a^2}\cos(ax) + \frac{1}{a}x\sin(ax) + C$$

(4.15) $$\int x^2\cos(ax)dx = \frac{2}{a^2}x\cos(ax) + \frac{1}{a}x^2\sin(ax) - \frac{2}{a^3}\sin(ax) + C$$

(4.16) $$\int x^n \cos x dx = x^n \sin x - n\int x^{n-1}\sin x dx$$

(4.17) $$\int \cos(ax)\sin(bx)dx = \frac{1}{2(a-b)}\cos[(a-b)x] - \frac{1}{2(a+b)}\cos[(a+b)x] + C$$

(4.18) $$\int \sin^2(ax)\cos(bx)dx = -\frac{1}{4(2a-b)}\sin[(2a-b)x] + \frac{1}{2b}\sin(bx) - \frac{1}{4(2a+b)}\sin[(2a+b)x] + C$$

(4.19) $$\int \sin^2(ax)\cos(ax)dx = \frac{1}{3a}\sin^3(ax) + C$$

(4.20) $$\int \cos^2(ax)\sin(ax)dx = -\frac{1}{3a}\cos^3(ax) + C$$

(4.21) $$\int \cos^2(ax)\sin(bx)dx = \frac{1}{4(2a-b)}\cos[(2a-b)x] - \frac{1}{2b}\cos(bx) - \frac{1}{4(2a+b)}\cos[(2a+b)x] + C$$

(4.22)

$$\int \sin^2(ax)\cos^2(bx)dx = \frac{1}{4}x - \frac{1}{8a}\sin(2ax) - \frac{1}{16(a-b)}\sin[2(a-b)x] + \frac{1}{8b}\sin(2bx) - \frac{1}{16(a+b)}\sin[2(a+b)x] + C$$

(4.23) $$\int \sin^2(ax)\cos^2(ax)dx = \frac{1}{8}x - \frac{1}{32a}\sin(4ax) + C$$

(4.24) $$\int \sin^m x\cos^n x = \begin{cases} -\dfrac{\sin^{m-1} x\cos^{n+1} x}{m+n} + \dfrac{m-1}{m+n}\displaystyle\int \sin^{m-2} x\cos^n x dx, m \neq -n \\ \dfrac{\sin^{m+1} x\cos^{n-1} x}{m+n} + \dfrac{n-1}{m+n}\displaystyle\int \sin^m x\cos^{n-2} x dx, m \neq -n \end{cases}$$

(4.25) $$\int \frac{1}{1\pm\sin x}dx = \tan x \mp \sec x + C$$

(4.26) $$\int \frac{dx}{1\pm\sin ax} = \mp\frac{1}{a}\tan\left(\frac{\pi}{4}\mp\frac{ax}{2}\right) + C$$

(4.27) $$\int \frac{1}{\sin x\cos x}dx = \int \csc x\sec x dx = \ln\left|\tan x\right| + C$$

(4.28) $$\int \frac{1}{1\pm\cos x}dx = -\cot x \pm \csc x + C$$

(4.29) $$\int \frac{dx}{1+\cos ax} = \frac{1}{a}\tan\left(\frac{ax}{2}\right) + C$$

(4.30) $$\int \frac{dx}{1-\cos ax} = -\frac{1}{a}\cot\left(\frac{ax}{2}\right) + C$$

(4.31) $$\int \tan ax dx = -\frac{1}{a}\ln|\cos ax| + C$$

(4.32) $$\int \tan^2 ax dx = \frac{1}{a}\tan ax - x + C$$

(4.33) $$\int \tan^3(ax)dx = \frac{1}{a}\ln|\cos(ax)| + \frac{1}{2a}\sec^2(ax) + C$$

(4.34) $$\int \tan^n x dx = \frac{1}{n-1}\tan^{n-1} x - \int \tan^{n-2} x dx, n \neq 1$$

(4.35) $$\int \frac{1}{1 \pm \tan x} dx = \frac{1}{2}\left[x \pm \ln|\cos x \pm \sin x|\right] + C$$

(4.36) $$\int \sec(ax)dx = \frac{1}{a}\ln|\sec(ax) + \tan(ax)| + C = \frac{2}{a}\tanh^{-1}\left(\tan\frac{x}{2}\right) + C$$

(4.37) $$\int \sec^2(ax)dx = \frac{1}{a}\tan(ax) + C$$

(4.38) $$\int \sec^3 x dx = \frac{1}{2}\sec x \tan x + \frac{1}{2}\ln|\sec x + \tan x| + C$$

(4.39) $$\int \sec^n x dx = \frac{1}{n-1}\sec^{n-2} x \tan x + \frac{n-2}{n-1}\int \sec^{n-2} x dx, n \neq 1$$

(4.40) $$\int \sec^2(ax)\tan(ax)dx = \frac{1}{2a}\sec^2(ax) + C$$

(4.41) $$\int \sec^n(ax)\tan(ax)dx = \frac{1}{na}\sec^n(ax) + C, n \neq 0$$

(4.42) $$\int \frac{1}{1 \pm \sec x} dx = x + \cot x \mp \csc x + C$$

(4.43) $$\int \cot^2(x)dx = -x - \cot x + C$$

(4.44) $$\int \cot^n x dx = -\frac{1}{n-1}\cot^{n-1} x - \int \cot^{n-2} x dx, n \neq 1$$

(4.45) $$\int \frac{1}{1 \pm \cot x} dx = \frac{1}{2}[x \mp \tan x \pm \sec x] + C$$

(4.46) $$\int \csc(ax)dx = \frac{1}{a}\ln\left|\tan\frac{ax}{2}\right| = \frac{1}{a}\ln|\csc(ax) - \cot(ax)| + C$$

(4.47) $$\int \csc^2(ax)dx = -\frac{1}{a}\cot(ax) + C$$

(4.48) $$\int \csc^3 x dx = \frac{1}{2}\ln|\csc x - \cot x| - \frac{1}{2}\csc x \cot x + C$$

(4.49) $$\int \csc^n x dx = -\frac{1}{n-1}\csc^{n-2} x \cot x + \frac{n-2}{n-1}\int \csc^{n-2} x dx, n \neq 1$$

(4.50) $$\int \csc^n(ax)\cot(ax)dx = -\frac{1}{na}\csc^n(ax) + C, n \neq 0$$

(4.51) $$\int \frac{1}{1 \pm \csc x}dx = x - \tan x \pm \sec x + C$$

Inverse Trigonometric Functions

(5.1) $$\int \arcsin x dx = x \arcsin x + \sqrt{1-x^2} + C$$

(5.2) $$\int x \arcsin x dx = \frac{1}{2}x^2 \arcsin x - \frac{1}{4}\arcsin x + \frac{1}{4}x\sqrt{1-x^2} + C$$

(5.3) $$\int x^n \arcsin x dx = \frac{1}{n+1}x^{n+1}\arcsin x - \frac{1}{n+1}\int \frac{x^{n+1}}{\sqrt{1-x^2}}dx + C, n \neq -1$$

(5.4) $$\int \arccos x dx = x \arccos x - \sqrt{1-x^2} + C$$

(5.5) $$\int x \arccos x dx = \frac{1}{2}x^2 \arccos x + \frac{1}{4}x\sqrt{1-x^2} + \frac{1}{4}\arcsin x + C$$

(5.6) $$\int \arctan x dx = x \arctan x - \ln \sqrt{1+x^2} + C$$

(5.7) $$\int x \arctan x dx = \frac{1}{2}x^2 \arctan x + \frac{1}{2}\arctan x - \frac{1}{2}x + C$$

(5.8) $$\int x^n \arctan x dx = \frac{1}{n+1}x^{n+1}\arctan x - \frac{1}{n+1}\int \frac{x^{n+1}}{x^2+1}dx + C, n \neq -1$$

(5.9) $$\int \operatorname{arccot} x dx = x \operatorname{arccot} x + \ln \sqrt{1+x^2} + C$$

(5.10) $$\int x \operatorname{arccot} x dx = \frac{1}{2}x^2 \operatorname{arccot} x + \frac{1}{2}x + \frac{1}{2}\operatorname{arccot} x + C$$

(5.11) $$\int \operatorname{arcsec} x dx = x \operatorname{arcsec} x - \ln\left|x + \sqrt{x^2-1}\right| + C$$

(5.12) $$\int x \operatorname{arcsec} x dx = \frac{1}{2}x^2 \operatorname{arcsec} x - \frac{1}{2}\sqrt{x^2-1} + C$$

(5.13) $$\int x^n \operatorname{arcsec} x dx = \frac{1}{n+1}x^{n+1}\operatorname{arcsec} x - \frac{1}{n+1}\int \frac{x^n}{\sqrt{x^2-1}}dx + C, n \neq -1$$

(5.14) $$\int \operatorname{arccsc} x dx = x \operatorname{arccsc} x + \ln\left|x + \sqrt{x^2-1}\right| + C$$

(5.15) $$\int x \operatorname{arccsc} x dx = \frac{1}{2}x^2 \operatorname{arccsc} x + \frac{1}{2}\sqrt{x^2-1} + C$$

Hyperbolic Trigonometric Functions

(6.1) $$\int \sinh(ax)dx = \frac{1}{a}\cosh(ax) + C$$

(6.2) $$\int \cosh(ax)dx = \frac{1}{a}\sinh(ax) + C$$

(6.3) $$\int \tanh(ax)dx = \frac{1}{a}\ln|\cosh(ax)| + C$$

(6.4) $$\int \coth(ax)dx = \frac{1}{a}\ln|\sinh(ax)| + C$$

(6.5) $$\int \text{sech}(ax)dx = \frac{2}{a}\arctan\left(e^{ax}\right) + C$$

(6.6) $$\int \text{csch}(ax)dx = \frac{1}{a}\ln\left|\frac{e^{ax}-1}{e^{ax}+1}\right| + C$$

(6.7) $$\int x\sinh(ax)dx = \frac{1}{a}x\cosh(ax) - \frac{1}{a^2}\sinh(ax) + C$$

(6.8) $$\int x\cosh(ax)dx = \frac{1}{a}x\sinh(ax) - \frac{1}{a^2}\cosh(ax) + C$$

(6.9) $$\int x^n\sinh(ax)dx = \frac{1}{a}x^n\cosh(ax) - \frac{n}{a}\int x^{n-1}\cosh(ax)dx + C$$

(6.10) $$\int x^n\cosh(ax)dx = \frac{1}{a}x^n\sinh(ax) - \frac{n}{a}\int x^{n-1}\sinh(ax)dx + C$$

(6.11) $$\int \sinh(ax)\cosh(ax)dx = \frac{1}{4a}\sinh(2ax) - \frac{1}{2}x + C$$

(6.12) $$\int \sinh(ax)\cosh(bx)dx = \frac{1}{b^2-a^2}\left[b\sinh(ax)\cosh(bx) - a\sinh(bx)\cosh(ax)\right] + C$$

(6.13) $$\int \sinh(ax)\sin(bx)dx = \frac{1}{a^2+b^2}\left[a\cosh(ax)\sin(bx) - b\sinh(bx)\cos(ax)\right] + C$$

(6.14) $$\int \sinh(ax)\cos(bx)dx = \frac{1}{a^2+b^2}\left[a\cosh(ax)\cos(bx) + b\sinh(ax)\sin(bx)\right] + C$$

(6.15) $$\int \cosh(ax)\sin(bx)dx = \frac{1}{a^2+b^2}\left[a\sinh(ax)\sin(bx) - b\cosh(ax)\cos(bx)\right] + C$$

(6.16) $$\int \cosh(ax)\cos(bx)dx = \frac{1}{a^2+b^2}\left[b\cosh(ax)\sin(bx) + a\sinh(ax)\cos(bx)\right] + C$$

Logarithmic Functions

(7.1) $$\int \ln(ax)dx = x\ln(ax) - x + C$$

(7.2) $$\int \frac{\ln(ax)}{x}dx = \frac{1}{2}\ln^2(ax) + C$$

(7.3) $$\int x^n\ln x dx = -\frac{1}{(n+1)^2}x^{n+1} + \frac{1}{n+1}x^{n+1}\ln x + C, n \neq -1$$

(7.4) $$\int \ln(ax+b)dx = \left(x + \frac{b}{a}\right)\ln(ax+b) - x, a \neq 0$$

(7.5) $$\int \ln(x^2+a^2)dx = x\ln(x^2+a^2) + 2a\arctan\left(\frac{x}{a}\right) - 2x + C$$

(7.6) $$\int \ln(x^2-a^2)dx = x\ln(x^2-a^2) + a\ln\left|\frac{x+a}{x-a}\right| - 2x + C$$

(7.7) $$\int \ln(ax^2+bx+c)dx = \frac{1}{a}\sqrt{4ac-b^2}\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right) - 2x + \left(\frac{b}{2a}+x\right)\ln(ax^2+bx+c) + C$$

(7.8) $$\int x\ln(ax+b)dx = \frac{b}{2a}x - \frac{1}{4}x^2 + \frac{1}{2}x^2\ln(ax+b) - \frac{b^2}{2a^2}\ln(ax+b) + C$$

(7.9) $$\int x\ln(a^2-b^2x^2)dx = -\frac{1}{2}x^2 + \frac{1}{2}x^2\ln(a^2-b^2x^2) - \frac{a^2}{2b^2}\ln(ax+b) + C$$

(7.10) $$\int \ln^2 x dx = 2x - 2x\ln x + x\ln^2 x + C$$

(7.11) $$\int \ln^n x dx = n\ln^n x - n\int \ln^{n-1} x dx + C$$

(7.12) $$\int x^n \ln^m x dx = \frac{1}{n+1}x^{n+1}\ln^m x - \frac{m}{n+1}\int x^n \ln^{m-1} x dx + C, m, n \neq -1$$

(7.13) $$\int \frac{1}{x\ln x}dx = \ln|\ln x| + C$$

(7.14) $$\int \log_a bx dx = \frac{1}{\ln a}[x\ln bx - x] + C$$

Exponential Functions

(8.1) $$\int e^{ax}dx = \frac{1}{a}e^{ax} + C$$

(8.2) $$\int xe^{ax}dx = \frac{1}{a}xe^{ax} - \frac{1}{a^2}e^{ax} + C = \frac{ax-1}{a^2}e^{ax} + C$$

(8.3) $$\int x^2e^{ax}dx = \frac{a^2x^2-2ax+2}{a^3}e^{ax} + C$$

(8.4) $$\int x^3e^{ax}dx = \frac{a^3x^3-3a^2x^2+6ax-6}{a^4}e^{ax} + C$$

(8.5) $$\int x^ne^{ax}dx = \frac{1}{a}x^ne^{ax} - \frac{n}{a}\int x^{n-1}e^{ax}dx + C$$

(8.6) $$\int \frac{e^{ax}}{x^n}dx = -\frac{e^{ax}}{(n-1)x^{n-1}} + \frac{a}{n-1}\int \frac{e^{ax}}{x^{n-1}}dx + C, n \neq 1$$

(8.7) $$\int xe^{-ax^2}dx = -\frac{1}{2a}e^{-ax^2} + C$$

(8.8) $$\int x^3e^{ax^2}dx = \frac{1}{2a}x^2e^{ax^2} - \frac{1}{2a^2}e^{ax^2} + C$$

(8.9) $$\int x^{2n+1}e^{ax^2}dx = \frac{1}{2a}x^{2n}e^{ax^2} - \frac{n}{a}\int x^{2n-1}e^{ax^2}dx + C, n \geq 0$$

(8.10) $$\int \frac{1}{a+be^{kx}}dx = \frac{1}{ak}\left[kx - \ln\left|a+be^{kx}\right|\right] + C$$

(8.11) $$\int e^{ax}\sin(bx)dx = \frac{e^{ax}}{a^2+b^2}\left[a\sin(bx) - b\cos(bx)\right] + C$$

(8.12) $$\int e^{ax}\cos(bx)dx = \frac{e^{ax}}{a^2+b^2}\left[b\sin(bx) + a\cos(bx)\right] + C$$

(8.13) $$\int xe^{x}\sin x dx = \frac{1}{2}e^{x}\left[\cos x - x\cos x + x\sin x\right] + C$$

(8.14) $$\int xe^{x}\cos x dx = \frac{1}{2}e^{x}\left[x\cos x + x\sin x - \sin x\right] + C$$

(8.15) $$\int e^{ax}\sinh(bx)dx = \begin{cases} \frac{e^{ax}}{a^2-b^2}\left[a\sinh(bx) - b\cosh(bx)\right] + C, a^2 \neq b^2 \\ \frac{1}{4a}e^{2ax} - \frac{x}{2} + C, a = b \end{cases}$$

(8.16) $$\int e^{ax}\cosh(bx)dx = \begin{cases} \frac{e^{ax}}{a^2-b^2}\left[a\cosh(bx) - b\sinh(bx)\right] + C, a^2 \neq b^2 \\ \frac{1}{4a}e^{2ax} + \frac{x}{2} + C, a = b \end{cases}$$

(8.17) $$\int a^{bx}dx = \frac{1}{b\ln a}a^{bx} + C, a > 0, a \neq 1$$

MAT 192, Exam #1, Part II, Fall 2016 Name ______________________________

Instructions: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

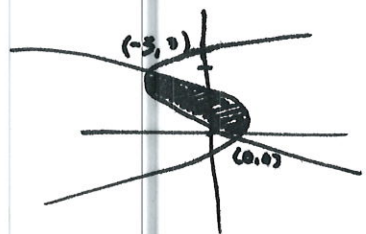
1. Find the area of the region bounded by $x = y^2 - 4y$ and $x = 2y - y^2$. Sketch the region.

$y^2 - 4y = 2y - y^2$

$+y^2 + 4y \quad +4y \quad +y^2$

$2y^2 = 6y \qquad y = 0$

$y = 3$



$$\int_0^3 2y - y^2 - (y^2 - 4y)\,dy = \int_0^3 6y - 2y^2\,dy = 3y^2 - \frac{2}{3}y^3 \Big|_0^3 =$$

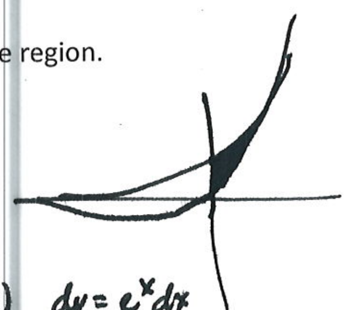
$$27 - \frac{2}{3}(27) = 27 - 18 = \boxed{9}$$

2. Find the area bounded by $y = e^x, y = xe^x, x = 0$. Sketch the region.

$e^x = xe^x$

$0 = xe^x - e^x = e^x(x-1)$

$x = 1$



$$\int_0^1 e^x - xe^x\,dx = \int_0^1 (1-x)e^x\,dx \qquad u = (1-x) \quad dv = e^x dx \quad du = -dx \quad v = e^x$$

$$(1-x)e^x + \int e^x\,dx = (1-x)e^x + e^x \quad +C$$

$$= -xe^x + 2e^x \Big|_0^1 = -(1)e^1 + 2e^1 + 0e^0 - 2e^0 =$$

$$\boxed{e - 2}$$

3. Use integration by parts to evaluate $\int \arcsin x\, dx$.

$u = \arcsin x \qquad dv = dx$

$du = \frac{1}{\sqrt{1-x^2}}dx \qquad v = x$

$$x\arcsin x - \int \frac{x}{\sqrt{1-x^2}}\,dx$$

$u = 1 - x^2$

$du = -2x\,dx$

$-\frac{1}{2}du = x\,dx$

$\int u^{-1/2}du$

$$x\arcsin x + \frac{1}{2}\cdot 2u^{1/2} + C$$

$$\boxed{x\arcsin x + \sqrt{1-x^2} + C}$$

4. Use the tabular method to evaluate $\int z^3 e^z \, dz$.

$z^3e^z - 3z^2e^z + 6ze^z - 6e^z + C$

	u	dv
+	z^3	e^z
−	$3z^2$	e^z
+	$6z$	e^z
−	6	e^z
	0	e^z

5. Set up the integral $\int \sin^7\theta \cos^5\theta \, d\theta$ with a substitution so that it can be integrated as a polynomial. You do not need to complete the integration.

if $u = \sin\theta$
$du = \cos\theta \, d\theta$

$\int \sin^7\theta \cos^4\theta \cdot \cos\theta \, d\theta$

$\int \sin^7\theta (1-\sin^2\theta)^2 \cos\theta \, d\theta$

$\int u^7 (1-u^2)^2 \, du$

or

$u = \cos\theta$
$-du = +\sin\theta \, d\theta$

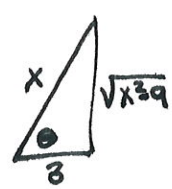
$\int \sin\theta (\sin^2\theta)^3 \cos^5\theta \, d\theta$

$\int \sin\theta (1-\cos^2\theta)^3 \cos^5\theta \, d\theta$

$-\int (1-u^2)^3 u^5 \, du$

6. Use trig substitution to evaluate $\int \frac{\sqrt{x^2-9}}{x^3} dx$.

$x = 3\sec\theta$
$dx = 3\sec\theta\tan\theta \, d\theta$



$\int \frac{3\tan\theta \cdot 3\sec\theta\tan\theta}{27\sec^3\theta} d\theta = \frac{1}{3}\int \frac{\tan^2\theta}{\sec^2\theta} d\theta$

$x^2 = 9\sec^2\theta$

$x^2 - 9 = 9\sec^2\theta - 9 = 9(\tan^2\theta)$
$9(\sec^2\theta - 1)$

$\frac{1}{3}\int \frac{\sec^2\theta - 1}{\sec^2\theta} d\theta = \frac{1}{3}\int 1-\cos^2\theta \, d\theta = \frac{1}{3}\int \sin^2\theta \, d\theta$

$\sqrt{x^2-9} = 3\tan\theta$

$= \frac{1}{6}\int 1-\cos 2\theta \, d\theta = \frac{1}{6}\left[\theta - \frac{1}{2}\sin 2\theta\right] + C = \boxed{\frac{1}{6}\operatorname{arcsec}\left(\frac{x}{3}\right) - \frac{\sqrt{x^2-9}}{2x^2} + C}$

$\sin\theta\cos\theta$

7. Use partial fractions to integrate $\int \frac{1}{t^3-1} dt$.

$t^3-1 = (t-1)(t^2+t+1)$

$\frac{A}{t-1} + \frac{Bt+C}{t^2+t+1} = \frac{At^2+At+A+Bt^2-Bt+Ct-C}{t^3-1} = \frac{1}{t^3-1}$

$A+B=0 \Rightarrow A=-B$

$A-B+C=0 \Rightarrow 2A+C=0$

$A-C=1$; $3A=1$

$A=1/3 \quad B=-1/3 \quad C=-2/3$

$\frac{1}{3}\int \frac{1}{t-1} dt + \frac{1}{3}\int \frac{-t-2}{t^2+t+1} dt$

$\frac{1}{3}\int \frac{1}{t-1} - \frac{1}{3}\int \frac{t+1}{t^2+t+1} dt = \frac{1}{3}\int \frac{1}{t-1} dt - \frac{1}{6}\left[\frac{2t+1}{t^2+t+1} + \frac{+1}{(t+1/2)^2+\frac{3}{4}}\right]$

$(t^2+t+1/4)+\frac{3}{4}$

$$\frac{1}{3}\ln|t-1| - \frac{1}{6}\left[\ln|t^2+t+1| + \frac{2}{\sqrt{3}}\arctan\left(\frac{(t+1/2)2}{\sqrt{3}}\right)\right] + C$$

8. Which method of integration should be used to evaluate each integral? You do not need to integrate. If you are using a substitution method, state the substitution used.

a. $\int \cos x \ln(\sin x)\, dx$

$\int \ln u \, du$ $\quad u = \sin x$, $du = \cos x$

by parts

e. $\int \cos^2\theta \sin^2\theta \, d\theta$

$\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$

$\sin^2\theta = \frac{1}{2}(1-\sin 2\theta)$

identities

(substitution at end)

b. $\int x^5 e^{-x^3} dx$

by parts

f. $\int \sqrt{x^2+2x}\,dx$

trig substitution

(complete square first)

c. $\int \frac{t}{t^4+2} dt$

Substitution for t^2

arctan rule

g. $\int \cot x \ln(\sin x)\, dx$

Substitution

$u = \ln(\sin x)$

$du = \cot x \, dx$

d. $\int \frac{x^3-4x-10}{x^2-x-6} dx$

long division

then partial fractions

h. $\int \frac{x^3+2x}{x^4+4x^2+3} dx$

partial fractions

(substitution)

9. Integrate.

a. $\int \sin 8x \cos 5x \, dx$ $= \frac{1}{2}\int \sin 13x + \sin 3x \, dx$

$-\frac{1}{26}\cos 13x - \frac{1}{6}\cos 3x + C$

b. $\int \frac{dx}{\cos x - 1} \cdot \frac{\cos x + 1}{\cos x + 1} = \int \frac{\cos x + 1}{\cos^2 x - 1} dx = \int \frac{\cos x + 1}{\sin^2 x} dx = \int \cot x \csc x + \csc^2 x \, dx$

$+\csc x - \cot x + C$

c. $\int \sin^2 x \, dx$ $= \frac{1}{2}\int (1 - \cos 2x) \, dx$

$\frac{1}{2}\left[x - \frac{1}{2}\sin 2x\right] + C$

d. $\int e^{\cos t} \sin 2t \, dt$ $= 2\int e^{\cos t} \sin t \cos t \, dt$ $\quad u = \cos t$, $du = -\sin t \, dt$

$-2\int e^u \cdot u \, du = -2\left[ue^u - e^u\right] + C$

$= -2\cos t \, e^{\cos t} + 2e^{\cos t} + C$

+	u	e^u
−	1	e^u
	0	e^u

10. Apply partial fractions to the expression $\frac{1}{(x^2-4)(x^2-x-6)(x^2+1)x^3}$. You do not need to integrate or solve for the coefficients.

$(x-2)(x+2)(x-3)(x+2)$ → $(x+2)^2$

$\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{x-2} + \frac{E}{(x+2)} + \frac{F}{(x+2)^2} + \frac{G}{(x-3)} + \frac{Hx+I}{x^2+1}$

11. Use the Trapezoidal Rule with $n = 6$ to estimate $\int_4^6 \ln(x^3 + 2)\, dx$.

$\frac{b-a}{n} = \frac{6-4}{6} = \frac{2}{6} = \frac{1}{3}$

$$f(x) \approx \frac{1}{6}\left[\ln(4^3+2) + 2\ln\left(\left(\tfrac{13}{3}\right)^3+2\right) + 2\ln\left(\left(\tfrac{14}{3}\right)^3+2\right) + 2\ln\left((5)^3+2\right) + 2\ln\left(\left(\tfrac{16}{3}\right)^3+2\right) + 2\ln\left(\left(\tfrac{17}{3}\right)^3+2\right) + \ln(6^3+2)\right] \approx$$

9.648379816...

12. Use Simpson's Rule to estimate $\int_0^2 \frac{1}{1+x^5}\, dx$ with $n = 4$.

$\frac{2-0}{4} = \frac{2}{4} = \frac{1}{2}$

$$f(x) = \frac{1}{6}\left[\frac{1}{1+0^5} + \frac{4}{1+(1/2)^5} + \frac{2}{1+1^5} + \frac{4}{1+(3/2)^5} + \frac{1}{1+2^5}\right] \approx$$

1.062424242...

13. If we wanted to estimate $\int_0^1 \sin x^2\, dx$ to within $E \leq 0.001$ using Simpson's Rule. What size n would be needed.

$$.0001 \geq \frac{(1-0)^5}{180n^4}(.76261458)$$

$$n^4 \geq \frac{.76261458}{180} \cdot 10{,}000$$

$$n^4 \geq 40.3478\ldots$$

$$n \geq 2.52$$

$\Rightarrow n = 4$

(must be an even # and round up)

$f(x) = \sin x^2$

$f'(x) = 2x\cos x^2$

$f''(x) = 2\cos x^2 - 4x^2\sin x^2$

$f'''(x) = -4x\sin x^2 - 8x\sin x^2 - 8x^3\cos x^2$

$= -12x\sin x^2 - 8x^3\cos x^2$

$f^{iv}(x) = -12\sin x^2 - 24x^2\cos x^2 - 24x^2\cos x^2 + 16x^4\sin x^2$

$= -12\sin x^2 - 48x^2\cos x^2 + 16x^4\sin x^2$

$\max|f^{iv}(x)| \approx .76261458$

14. Determine if the improper integrals converge or diverge. Sketch the graph of the region to find all points of discontinuity insider the interval. If it converges, evaluate it.

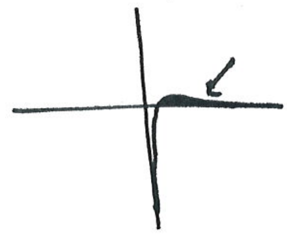
a. $\int_1^{\infty} \frac{\ln x}{x} dx$

$u = \ln x$ $\quad \int u \, du = \frac{1}{2}u^2 + c$

$du = \frac{1}{x} dx$

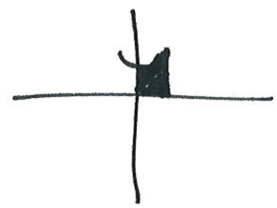
$\lim_{b \to \infty} \frac{1}{2} \ln^2 x \Big|_1^b = \lim_{b \to \infty} \frac{1}{2}\left[\ln^2 b - \ln^2(1)\right] = \infty$

diverges



b. $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$

$f(x)$ not defined at $x = 1$



$\lim_{b \to 1} \arcsin x \Big|_0^b = \lim_{b \to 1} \arcsin b - \arcsin 0$

$= \arcsin 1 = \pi/2$

Converges

Some useful formulas:

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$\cos^2 t = \frac{1}{2}(1 + \cos 2t)$$

$$\sin 2t = 2 \sin t \cos t \,, \cos 2t = \cos^2 t - \sin^2 t$$

$$\cos a \cos b = \frac{1}{2}[\cos(a+b) + \cos(a-b)]$$

$$\sin a \sin b = \frac{1}{2}[\cos(a-b) - \cos(a+b)]$$

$$\sin a \cos b = \frac{1}{2}[\sin(a+b) + \sin(a-b)]$$

Trapezoidal Rule Error: $|E| \leq \frac{(b-a)^3}{12n^2}[\max|f''(x)|]$

Simpson's Rule Error: $|E| \leq \frac{(b-a)^5}{180n^4}[\max|f^{IV}(x)|]$