

Instructions: Show all work. Answers without work will be worth only one point per problem. Give exact answers (fractions!) unless specifically asked to round. Complete all parts of each problem.

1. Simplify.

$$a. \frac{7}{8} \div \frac{21}{32} = \frac{\cancel{7}^1}{\cancel{8}_2} \cdot \frac{3\cancel{2}^4}{\cancel{21}_3} = \frac{4}{3}$$

$$b. \frac{7}{15} + \frac{5}{6} = \frac{7}{15} \cdot \frac{2}{2} + \frac{5}{6} \cdot \frac{5}{5} = \frac{14}{30} + \frac{25}{30} = \frac{39}{30} = \frac{13}{10}$$

$$c. 5[3(2+5) - 5] = 5[3(7) - 5] = 5[21 - 5] = 5(16) = 80$$

$$d. \frac{|5-9| + |10-15|}{|2(-3)|} = \frac{|-4| + |-5|}{|-6|} = \frac{4+5}{6} = \frac{9}{6} = \frac{3}{2}$$

$$e. \frac{6+(-2)^2}{4-9} = \frac{6+4}{-5} = \frac{10}{-5} = -2$$

$$f. \frac{-15+(-4)^2+|-9|}{10-2 \cdot 5} = \frac{-15+16+9}{10-10} = \frac{10}{0} \text{ undefined}$$

2. Evaluate $\frac{y+z-1}{x}$ for $x=6, y=-2, z=-3$

$$\frac{-2+(-3)-1}{6} = \frac{-6}{6} = -1$$

3. Simplify each expression as much as possible.

a. $-4(3x+5) - 7$

$$-12x - 20 - 7 = -12x - 27$$

b. $5g - 3 - 5 - 5g$

$$-8$$

c. $4(2x-3) - 2(x+1)$

$$8x - 12 - 2x - 2 = 6x - 14$$

$$d. 14 - 11(5m + 3n) = 14 - 55m - 33n$$

$$e. \frac{1}{3}(9x - 6) - (x - 2) = 3x - 2 - x + 2 = 2x$$

4. Translate each English statement into an algebraic expression.

a. Twice a number decreased by four.

$$2x - 4$$

b. Eight times the sum of a number and six.

$$8(x + 6)$$

5. Solve each equation. Clearly state the solution.

$$a. \frac{x}{3} + 2 = -5$$

$$\frac{3}{1} \left(\frac{x}{3} - 2 \right) \cdot \frac{3}{1} \Rightarrow \boxed{x = -21}$$

$$b. -3 = -5(4x + 3) + 21x$$

$$\begin{aligned} -3 &= -20x - 15 + 21x \\ -3 &= -15 + x \\ +15 \quad +15 & \\ \hline 12 &= x \end{aligned} \quad \Rightarrow \boxed{x = 12}$$

$$c. 8 - 2(a + 1) = 9 + a$$

$$\begin{aligned} 8 - 2a - 2 &= 9 + a \\ 6 - 2a &= 9 + a \\ +2a \quad +2a & \\ \hline 6 &= 9 + 3a \\ -9 \quad -9 & \\ \hline -3 &= 3a \\ \frac{-3}{3} &= \frac{3a}{3} \end{aligned} \quad \Rightarrow \boxed{a = -1}$$

$$d. \left(\frac{2(x+1)}{4} = 3x-2 \right) \frac{4}{4} \Rightarrow 2(x+1) = 4(3x-2) \Rightarrow 2x+2 = 12x-8$$

$$\begin{array}{r} -2x \quad -2x \\ \hline 2 = 10x - 8 \\ +8 \quad +8 \\ \hline 10 = 10x \\ \Rightarrow \boxed{x=1} \end{array}$$

$$e. \left(\frac{4(5-w)}{3} = -w \right) \frac{3}{3} \Rightarrow 4(5-w) = -3w$$

$$\begin{array}{r} 20-4w = -3w \\ +4w \quad +4w \\ \hline 20 = w \end{array}$$

$$\boxed{w=20}$$

$$f. \left(\frac{7}{8}x + \frac{1}{4} = \frac{3}{4}x \right) \frac{8}{8} \Rightarrow \frac{7}{1} \cdot \frac{7}{8}x + \frac{2}{1} \cdot \frac{1}{4} = \frac{2}{1} \cdot \frac{3}{4}x$$

$$\begin{array}{r} 7x+2 = 6x \\ -6x \quad -6x \\ \hline x+2=0 \\ -2 \quad -2 \\ \hline \end{array}$$

$$\boxed{x=-2}$$

$$g. -5(4y-3) + 2 = -20y + 17$$

$$-20y + 15 + 2 = -20y + 17$$

$$-20y + 17 = -20y + 17$$

x is all real #'s

$$h. 4(2+x) + 1 = 7x - 3(x-2)$$

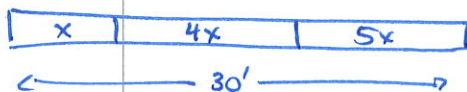
$$8+4x+1 = 7x-3x+6$$

$$\begin{array}{r} 9+4x = 4x+6 \\ -4x \quad -4x \\ \hline 9 = 6 \end{array}$$

$$9 = 6$$

x has no solution

6. A 30-foot piece of siding is cut into three pieces so that the second piece is four times as long as the first piece, and the third piece is five times as long as the first piece. If x represents the length of the first piece, find the lengths of all three pieces.



$$x + 4x + 5x = 30$$

$$\frac{10x}{10} = \frac{30}{10} \Rightarrow x=3$$

shortest piece = 3 feet
middle piece = 12 feet
longest piece = 15 feet

7. The code to unlock a student's lock happens to be three consecutive numbers that sum to 51. Find the integers that open the lock.

$$\begin{array}{r} x + x + 1 + x + 2 = 51 \\ 3x + 3 = 51 \\ \underline{-3 \quad -3} \\ 3x = 48 \end{array}$$

$$\frac{3x}{3} = \frac{48}{3}$$

$$x = 16$$

lock combination is 16, 17, 18

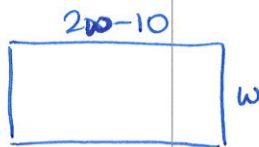
8. Solve the equation $A = P + PRT$ for T .

$$\frac{A-P}{PR} = \frac{PRT}{PR} \Rightarrow \frac{A-P}{PR} = T$$

9. Solve the equation $V = \frac{1}{3}hr^2$ for h .

$$\frac{3V}{r^2} = \frac{hr^2}{r^2} \Rightarrow \frac{3V}{r^2} = h$$

10. If the length of a rectangular parking lot is 10 meters less than twice its width, and the perimeter is 400 meters, find the dimensions of the lot.



$$2(2w-10) + 2w = 400$$

$$4w - 20 + 2w = 400$$

$$\begin{array}{r} 6w - 20 = 400 \\ +20 \quad +20 \\ \hline 6w = 420 \end{array}$$

$$\frac{6w}{6} = \frac{420}{6}$$

$$w = 70$$

lot is 70 by 130 m.

$$l = 2(70) - 10 = 140 - 10 = 130$$

11. Find the original price of a popular pair of shoes if the price is now \$78 after a 25% discount.

$$x - .25x = 78$$

$$\frac{.75x}{.75} = \frac{78}{.75}$$

$$x = \$104 \text{ before discount}$$

12. The owner of a local chocolate shop wants to develop a new trail mix. How many pounds of chocolate-covered peanuts worth \$5/lbs. should be mixed with 10 lbs. of granola bites with \$2/lbs. to get a mixture worth \$3/lbs.?

$$\begin{array}{r}
 x \cdot 5 + 10 \cdot 2 = (x+10)3 \\
 5x + 20 = 3x + 30 \\
 \underline{-3x} \qquad \qquad \underline{-3x} \\
 2x + 20 = 30 \\
 \underline{-20} \qquad \underline{-20} \\
 2x = 10
 \end{array}$$

$$\frac{2x}{2} = \frac{10}{2}$$

$$\boxed{x = 5 \text{ pounds}}$$

13. Bruce Blossom invested a sum of money at 10% annual simple interest and twice that amount at 12% annual simple interest. If his total yearly income from both investments was \$2890, how much was invested at each rate?

x at 10%
 $2x$ at 12%

$$\begin{array}{r}
 .10x + .12(2x) = 2890 \\
 .10x + .24x = 2890 \\
 \underline{.34x} = \underline{2890} \\
 \qquad \qquad \underline{.34} \qquad \underline{.34}
 \end{array}$$

$$\boxed{\begin{array}{l} \$x = 8500 \text{ at } 10\% \\ \$17,000 \text{ at } 12\% \end{array}}$$

14. A collection of dimes and quarters is retrieved from a soft drink machine. There are five times as many dimes as quarters. The total value of the coins is \$27.75. Find the number of dimes, and the number of quarters.

x = # of quarters
 $5x$ = # of dimes

$$\begin{array}{r}
 .10(5x) + .25(x) = 27.75 \\
 .50x + .25x = 27.75 \\
 \underline{.75x} = \underline{27.75} \\
 \qquad \underline{.75} \qquad \underline{.75}
 \end{array}$$

$$\boxed{\begin{array}{l} x = 37 \text{ quarters} \\ 185 \text{ dimes} \end{array}}$$

15. Two cars leave Las Vegas, Nevada at the same time after visiting the Las Vegas Motor Speedway. The cars travel in opposite directions, one traveling northeast at 65 mph and the other traveling southwest at 41 mph. When will the two cars be 530 miles apart?

$$d = r_1 t_1 + r_2 t_2$$

$$530 = 65t + 41t$$

$$\frac{530}{106} = \frac{106t}{106}$$

$$\boxed{t = 5 \text{ hours}}$$

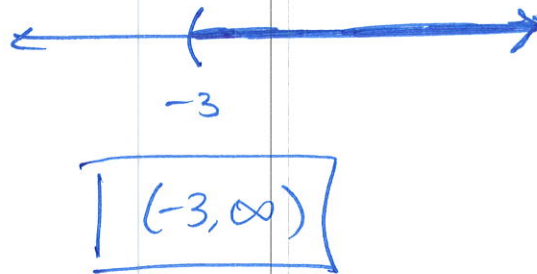
16. Solve the inequality. Draw the solution on a number line, and write the solution in interval notation.

a. $3x - 7 < 6x + 2$

$$\begin{array}{r} -6x \quad -6x \\ \hline -3x - 7 < 2 \\ +7 \quad +7 \\ \hline \end{array}$$

$$\begin{array}{r} -3x < 9 \\ \hline -3 \quad -3 \\ \hline \end{array}$$

$x > -3$ *flip neg.*

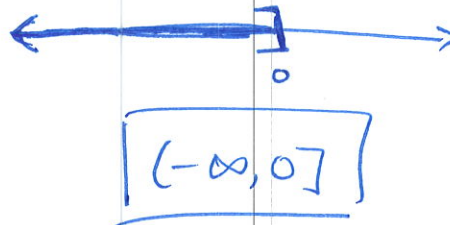


b. $6(2 - x) \geq 12$

$$\begin{array}{r} 12 - 6x \geq 12 \\ -12 \quad -12 \\ \hline \end{array}$$

$$\begin{array}{r} -6x \geq 0 \\ \hline -6 \quad -6 \\ \hline \end{array}$$

$x \leq 0$ *flip neg.*



c. $\frac{1}{4}(x + 4) < \frac{1}{5}(2x + 3)$

$$\frac{5}{1} \cdot \frac{1}{4}(x + 4) < \frac{4}{1} \cdot \frac{1}{5}(2x + 3)$$

$$5(x + 4) < 4(2x + 3)$$

$$\begin{array}{r} 5x + 20 < 8x + 12 \\ -8x \quad -8x \\ \hline -3x + 4 < 12 \end{array}$$

d. $-5 \leq 2(x + 4) < 8$

$$\begin{array}{r} -5 \leq 2x + 8 < 8 \\ -8 \quad -8 \quad -8 \\ \hline \end{array}$$

$$\begin{array}{r} -13 \leq 2x < 0 \\ \hline 2 \quad 2 \quad 2 \\ \hline \end{array}$$

$$-\frac{13}{2} \leq x < 0$$

$$\left[-\frac{13}{2}, 0 \right)$$

$$\begin{array}{r} -3x + 20 < 12 \\ -20 \quad -20 \\ \hline -3x < -8 \end{array}$$

$$\begin{array}{r} -3x < -8 \\ \hline -3 \quad -3 \\ \hline \end{array}$$

$x > \frac{8}{3}$ *flip neg.*

