Math 2568, Quiz #5, Fall 2014 Name KEY

Instructions: Show all work. Answer each question as completely as possible. Use exact values (yes, that means fractions!).

1. Determine if the following sets of vectors are linearly independent. Explain your reasoning.

   a. $\left\{\begin{bmatrix}1\\2\\3\end{bmatrix}, \begin{bmatrix}3\\2\\1\end{bmatrix}, \begin{bmatrix}1\\1\\1\end{bmatrix}\right\}$ $\Rightarrow \begin{bmatrix}1 & 0 & 1/4\\0 & 1 & 1/4\\0 & 0 & 0\end{bmatrix}$ not independent only 2 pivots

   b. $\left\{\begin{bmatrix}1\\-1\end{bmatrix}, \begin{bmatrix}1\\1\end{bmatrix}, \begin{bmatrix}1\\2\end{bmatrix}\right\}$ not independent too many vectors

   c. $\{t^3+t^2, t^3+t, t^2+t\}$ $\begin{bmatrix}0\\0\\1\\1\end{bmatrix}, \begin{bmatrix}0\\1\\0\\1\end{bmatrix}, \begin{bmatrix}0\\1\\1\\0\end{bmatrix}$ 3 pivots independent

   d. $\left\{\begin{bmatrix}1 & 0\\1 & 1\end{bmatrix}, \begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}, \begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}, \begin{bmatrix}0 & 0\\1 & 1\end{bmatrix}\right\}$ 4 pivots, independent

   $\begin{bmatrix}1\\0\\1\\1\end{bmatrix}, \begin{bmatrix}1\\1\\1\\1\end{bmatrix}, \begin{bmatrix}1\\1\\0\\1\end{bmatrix}, \begin{bmatrix}0\\0\\1\\1\end{bmatrix}$

2. For all the sets above that are linearly independent, do they form a basis for the space?
   a. For R^3 no
   b. For R^2 no
   c. For P_3 no
   d. For $M_{2\times2}$ yes

3. Find the dimension of the space spanned by the set.

   a. $H = \left\{\begin{bmatrix}2c\\a-b\\b-3c\\a+2b\end{bmatrix}\right\}$ $\begin{bmatrix}0\\1\\0\\1\end{bmatrix}a + \begin{bmatrix}0\\-1\\1\\2\end{bmatrix}b + \begin{bmatrix}2\\0\\-3\\0\end{bmatrix}c$ dim=3 (3 pivots)

   b. b. $M_{3\times4}$ 12

   c. c. Nul A and Col A of the matrix $A = \begin{bmatrix}1 & -6 & 9 & 0 & -2\\0 & 1 & 2 & -4 & 5\\0 & 0 & 0 & 5 & 1\\0 & 0 & 0 & 0 & 0\end{bmatrix}$

dim Nul A = 2

dim Col A = rank A = 3