

Instructions: Show all work. Answer each question as completely as possible. Use exact values (yes, that means fractions!).

1. Find the linear transformation with the following properties.

- a. The 2x2 rotation matrix that rotates a vector by the angle $\frac{2\pi}{3}$.

$$\begin{bmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{bmatrix}$$

- b. The 2x2 projection matrix that maps a vector onto the x_2 axis.

$$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

- c. The 2x2 shear matrix that maps the vector $\begin{bmatrix} 1 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} k \\ 1 \end{bmatrix}$.

$$\begin{bmatrix} k & 0 \\ 1 & 1 \end{bmatrix}$$

- d. The 2x2 matrix that compresses x_1 by a factor of $\frac{1}{2}$ and stretches x_2 by a factor of 4.

$$\begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 4 \end{bmatrix}$$

2. Find the null space of the matrix below. Write the solution in the form $\text{span}\{v_1, v_2, \dots, v_k\}$.

$$A = \begin{bmatrix} 1 & -2 & 3 & -6 & 5 & 0 \\ 0 & 0 & 0 & 1 & 4 & -6 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 3 & 0 & 29 & 0 \\ 0 & 0 & 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$x_1 = 2x_2 - 3x_3 - 29x_5$$

$$x_2 = x_2$$

$$x_3 = x_3$$

$$x_4 = -4x_5$$

$$x_5 = x_5$$

$$x_6 = 0$$

$$\vec{x} = \begin{bmatrix} 2 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} x_2 + \begin{bmatrix} -3 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} x_3 + \begin{bmatrix} -29 \\ 0 \\ 0 \\ -4 \\ 1 \\ 0 \end{bmatrix} x_5$$