Math 2568, Quiz #3, Fall 2014 Name KEY

Instructions: Show all work. Answer each question as completely as possible. Use exact values (yes, that means fractions!).

1. Consider the matrix below as the echelon form for a coefficient matrix for a homogeneous system. Write the solution to the system in parametric form.

$$A = \begin{bmatrix} 1 & -2 & 3 & -6 & 5 & 0 \\ 0 & 0 & 0 & 1 & 4 & -6 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\text{rref} \Rightarrow \begin{bmatrix} 1 & -2 & 3 & 0 & 29 & 0 \\ 0 & 0 & 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad \vec{x} = \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} x_2 + \begin{bmatrix} -3 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} x_3 + \begin{bmatrix} -29 \\ 0 \\ 0 \\ -4 \\ 1 \\ 0 \end{bmatrix} x_5$$

$x_1 - 2x_2 + 3x_3 + 29x_5 = 0$

$x_4 + 4x_5 = 0$

$x_6 = 0$

2. Determine if the following are subspaces of a vector space. If it is, prove it. If it is not, find a counterexample.

a. $H = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : x^2 + y^2 \le 1 \right\}$ not a subspace

$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ in H, $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ also in H, but $\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ not in set H.

b. $W = \begin{bmatrix} 2s + 4t \\ 2s \\ 2s - 3t \\ 5t \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 2 \\ 0 \end{bmatrix} s + \begin{bmatrix} 4 \\ 0 \\ -3 \\ 5 \end{bmatrix} t$ is a subspace

$$\vec{u} + \vec{v} = \left(\begin{bmatrix} 2 \\ 2 \\ 2 \\ 0 \end{bmatrix} s + \begin{bmatrix} 4 \\ 0 \\ -3 \\ 5 \end{bmatrix} t \right) + \left(\begin{bmatrix} 2 \\ 2 \\ 2 \\ 0 \end{bmatrix} p + \begin{bmatrix} 4 \\ 0 \\ -3 \\ 5 \end{bmatrix} q \right) = \begin{bmatrix} 2 \\ 2 \\ 2 \\ 0 \end{bmatrix} (s+p) + \begin{bmatrix} 4 \\ 0 \\ -3 \\ 5 \end{bmatrix} (t+q)$$

$s+p$ & $t+q$ still real

$\vec{0}$ is $s, t = 0$ $\quad k\left(\begin{bmatrix} 2 \\ 2 \\ 2 \\ 0 \end{bmatrix} s + \begin{bmatrix} 4 \\ 0 \\ -3 \\ 5 \end{bmatrix} t \right) = \begin{bmatrix} 2 \\ 2 \\ 2 \\ 0 \end{bmatrix} (ks) + \begin{bmatrix} 4 \\ 0 \\ -3 \\ 5 \end{bmatrix} (kt)$ $\quad ks$, & kt real

c. $G = \left\{ \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \in M_{2\times 3} \mid c = e = f = 0 \right\}$ $\quad \begin{bmatrix} a & b & 0 \\ d & 0 & 0 \end{bmatrix}$ $\quad \vec{0}$ if $a, b, d = 0$

$$\begin{bmatrix} a & b & 0 \\ d & 0 & 0 \end{bmatrix} + \begin{bmatrix} g & h & 0 \\ i & 0 & 0 \end{bmatrix} = \begin{bmatrix} a+g & b+h & 0 \\ d+i & 0 & 0 \end{bmatrix}$$

$a+g$, $b+h$, $d+i$ still real

0's all still 0's

$$k\begin{bmatrix} a & b & 0 \\ d & 0 & 0 \end{bmatrix} = \begin{bmatrix} ka & kb & 0 \\ kd & 0 & 0 \end{bmatrix}$$

ka, kb, kd all real

other entries still 0,

still in G

is a subspace

3. Determine if the transformation $S: R^3 \to R^2, S\left(\begin{bmatrix} a \\ b \\ c \end{bmatrix}\right) = \begin{bmatrix} a + 2b - 3c \\ 0 \end{bmatrix}$ is linear or not. Is so, prove it. If not, find a counterexample.

$S(\vec{0}) = 0$ Since $S\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 0 + 2(0) - 3(0) \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ ✓

$$S\left(\begin{bmatrix} a \\ b \\ c \end{bmatrix} + \begin{bmatrix} d \\ e \\ f \end{bmatrix}\right) = S\left(\begin{bmatrix} a+d \\ b+e \\ c+f \end{bmatrix}\right) = \begin{bmatrix} (a+d) + 2(b+e) - 3(c+f) \\ 0 \end{bmatrix} =$$

$$\begin{bmatrix} (a + 2b - 3c) + (d + 2e - 3f) \\ 0 \end{bmatrix}$$

$$S\left(\begin{bmatrix} a \\ b \\ c \end{bmatrix}\right) + S\left(\begin{bmatrix} d \\ e \\ f \end{bmatrix}\right) = \begin{bmatrix} a + 2b - 3c \\ 0 \end{bmatrix} + \begin{bmatrix} d + 2e - 3f \\ 0 \end{bmatrix} = \begin{bmatrix} a + 2b - 3c + d + 2e - 3f \\ 0 \end{bmatrix}$$

checks ✓

$$S\left(k\begin{bmatrix} a \\ b \\ c \end{bmatrix}\right) = S\left(\begin{bmatrix} ka \\ kb \\ kc \end{bmatrix}\right) = \begin{bmatrix} ka + 2kb - 3kc \\ 0 \end{bmatrix}$$

$$kS\left(\begin{bmatrix} a \\ b \\ c \end{bmatrix}\right) = k\begin{bmatrix} a + 2b - 3c \\ 0 \end{bmatrix} = \begin{bmatrix} ka + 2kb - 3kc \\ 0 \end{bmatrix}$$ checks.

yes, the transformation is linear