Math 2568, Quiz #12, Fall 2014 Name KEY

Instructions: Show all work. Answer each question as completely as possible. Use exact values (yes, that means fractions!).

1. Find the steady-state vector of the stochastic matrix $A = \begin{bmatrix} .4 & .2 \\ .6 & .8 \end{bmatrix}$ by hand. You may verify your results in your calculator, but you must show work.

$\begin{matrix} -.6 & .2 \\ .6 & -.2 \end{matrix}$

$\frac{.6x_1}{.6} = \frac{.2x_2}{.6}$

$x_1 = \frac{1}{3}x_2$

$x_2 = x_2$

$\vec{x} = \begin{bmatrix} 1 \\ 3 \end{bmatrix} \Rightarrow \vec{q} = \begin{bmatrix} 1/4 \\ 3/4 \end{bmatrix}$

2. Solve the dynamical system $A = \begin{bmatrix} 1.7 & -0.3 \\ -1.2 & 0.8 \end{bmatrix}$. Classify the origin as an attractor, repeller or a saddle point. Sketch the eigenvectors on the graph, and plot several possible trajectories.

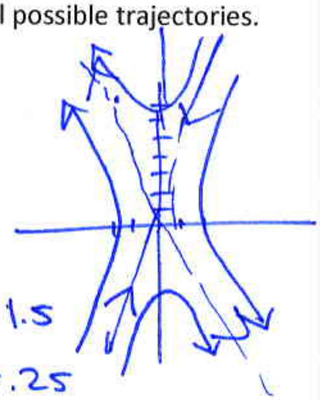
$(1.7-\lambda)(.8-\lambda) - (1.2)(.3) = 0$

$1.36 - 2.5\lambda + \lambda^2 - .36 = 0$

$\lambda^2 - 2.5\lambda + 1 = 0$

$\frac{2.5 \pm \sqrt{6.25-4}}{2} = \frac{2.5 \pm \sqrt{2.25}}{2} = 1.25 \pm 1.5$

$2.75, -.25$



origin is saddle point

$-1.05x_1 - .3x_2 = 0$

$x_1 = -2/7\, x_2$ $\begin{bmatrix} -2 \\ 7 \end{bmatrix}$

$1.95x_1 - .3x_2 = 0$

$x_1 = 2/13\, x_2$ $\begin{bmatrix} 2 \\ 13 \end{bmatrix}$

3. Solve the differential equation $\vec{x}' = \begin{bmatrix} -3 & 2 \\ -1 & -1 \end{bmatrix}\vec{x}$, by finding and plotting the eigenvectors, characterizing the origin as an attractor, repeller or saddle point, and writing the general solution of the system. Plot several sample trajectories on your graph.

$(-3-\lambda)(-1-\lambda) + 2 = 0$

$3 + 3\lambda + \lambda + \lambda^2 + 2 = 0$

$\lambda^2 + 4\lambda + 5 = 0$

$\begin{vmatrix} -3+2+i & 2 \\ -1 & -1+2+i \end{vmatrix} = \begin{vmatrix} -1+i & 2 \\ -1 & 1+i \end{vmatrix}$

$-x_1 + (1+i)x_2 = 0$

$(1+i)x_2 = x_1$

$x_2 = x_2$

$\vec{x} = \begin{bmatrix} 1+i \\ 1 \end{bmatrix}$ & $\begin{bmatrix} 1-i \\ 1 \end{bmatrix}$

spirals in

$\lambda = \frac{-4 \pm \sqrt{16-20}}{2} = -2 \pm i$

$e^{(-2\pm i)t} = e^{-2t}(\cos t + i\sin t)$ $c_1\begin{bmatrix} 1-i \\ 1 \end{bmatrix}e^{-2t}(\cos t + i\sin t) + c_2\begin{bmatrix} 1+i \\ 1 \end{bmatrix}e^{-2t}(\cos t - i\sin t)$