Math 2568, Quiz #10, Fall 2014 Name KEY

Instructions: Show all work. Answer each question as completely as possible. Use exact values (yes, that means fractions!).

1. Find the eigenvector associated with the eigenvalue $\lambda = 4$ for the matrix $A = \begin{bmatrix} 5 & 0 & -1 & 0 \\ 1 & 3 & 0 & 0 \\ 2 & -1 & 3 & 0 \\ 4 & -2 & -2 & 4 \end{bmatrix}$.

$$\begin{bmatrix} 1 & 0 & -1 & 0 \\ 1 & -1 & 0 & 0 \\ 2 & -1 & -1 & 0 \\ 4 & -2 & -2 & 0 \end{bmatrix} \xRightarrow{\text{rref}} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{array}{l} x_1 - x_3 = 0 \\ x_2 - x_3 = 0 \\ x_3 = x_3 \\ x_4 = x_4 \end{array}$$

$$\vec{x} = \begin{matrix} x_3 \\ x_3 \\ x_3 \\ x_4 \end{matrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix} x_3 + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} x_4$$

$$\left\{ \underbrace{\begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}}_{\vec{v}_{4,1}}, \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}}_{\vec{v}_{4,2}} \right\}$$

2. Find the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} -4 & 2 \\ 3 & 1 \end{bmatrix}$. Clearly indicate the characteristic polynomial.

$$\begin{vmatrix} -4-\lambda & 2 \\ 3 & 1-\lambda \end{vmatrix} = (-4-\lambda)(1-\lambda) - 6 = -4 + 4\lambda - \lambda + \lambda^2 - 6 =$$

$$-10 + 3\lambda + \lambda^2 = 0$$

$$\lambda^2 + 3\lambda - 10 = 0 \quad \text{Char. eq.}$$

$$(\lambda+5)(\lambda-2) = 0$$

$$\lambda = -5 \quad \lambda = 2$$

$$\begin{bmatrix} -4+5 & 2 \\ 3 & 1+5 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$$

$$x_1 + 2x_2 = 0$$

$$x_1 = -2x_2 \qquad \vec{v}_1 = \begin{bmatrix} -2 \\ 1 \end{bmatrix} x_2$$

$$x_2 = x_2$$

$$\begin{bmatrix} -4-2 & 2 \\ 3 & 1-2 \end{bmatrix} = \begin{bmatrix} -6 & 2 \\ 3 & -1 \end{bmatrix} \Rightarrow \begin{bmatrix} 3 & -1 \\ 0 & 0 \end{bmatrix}$$

$$3x_1 - x_2 = 0$$

$$3x_1 = x_2 \Rightarrow x_1 = \tfrac{1}{3}x_2 \Rightarrow \vec{v}_2 = \begin{bmatrix} 1 \\ 3 \end{bmatrix} x_2$$

$$x_2 = x_2$$