Name KEY

Math 2568, Final Exam – Part I, Fall 2014

Instructions: On this portion of the exam, you may **NOT** use a calculator. Show all work. Answers must be supported by work to receive full credit.

1. The system shown below is in vector equation form.

$$x_1\begin{bmatrix}4\\1\end{bmatrix}+x_2\begin{bmatrix}-1\\1\end{bmatrix}=\begin{bmatrix}5\\5\end{bmatrix}$$

a. Write the system as a matrix equation. (3 points)

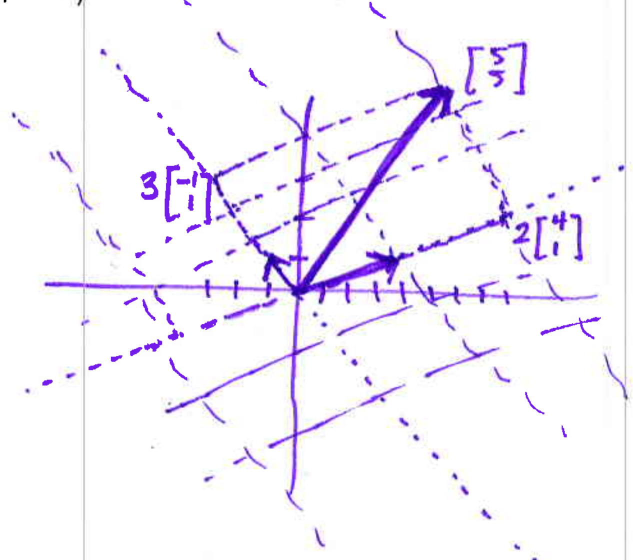
$$\begin{bmatrix}4 & -1\\1 & 1\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}5\\5\end{bmatrix}$$

b. Solve the system by using an inverse matrix. Write the solution as a column vector. (8 points)

$$A^{-1}=\frac{1}{4+1}\begin{bmatrix}1 & 1\\-1 & 4\end{bmatrix}=\begin{bmatrix}1/5 & 1/5\\-1/5 & 4/5\end{bmatrix}$$

$$A^{-1}\vec{b}=\begin{bmatrix}1/5 & 1/5\\-1/5 & 4/5\end{bmatrix}\begin{bmatrix}5\\5\end{bmatrix}=\begin{bmatrix}1+1\\-1+4\end{bmatrix}=\begin{bmatrix}2\\3\end{bmatrix}=\vec{x}$$

c. Use the solution you obtained and graphically represent it on a graph as the linear combination of the vectors $\begin{bmatrix}4\\1\end{bmatrix}$ and $\begin{bmatrix}-1\\1\end{bmatrix}$. Be sure to show the coordinate gridlines on your graph. (6 points)



2. Find the determinant of the matrix $A = \begin{bmatrix} 3 & 5 & 0 \\ 2 & -1 & 4 \\ 1 & 0 & 2 \end{bmatrix}$ by any means. (7 points)

$$3\begin{vmatrix} -1 & 4 \\ 0 & 2 \end{vmatrix} - 5\begin{vmatrix} 2 & 4 \\ 1 & 2 \end{vmatrix} = 3(-2+0) - 5(4-4) = -6$$

3. Find the QR factorization of the matrix A, given that $Q = \begin{bmatrix} 1/\sqrt{2} & -1/2 \\ 1/\sqrt{2} & 1/2 \\ 0 & 1 \end{bmatrix}$ $and\ A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}$. In other words, find R. (8 points)

$$R = Q^TA = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/2 & 1/2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} + 1/\sqrt{2} + 0 & 0 + 1/\sqrt{2} + 0 \\ -1/2 + 1/2 + 0 & 0 + 1/2 + 1 \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt{2} & 1/\sqrt{2} \\ 0 & 3/2 \end{bmatrix}$$

4. For the matrix $A = \begin{bmatrix} -5 & 10 & 4 & 14 \\ -7 & 11 & 5 & 13 \\ -3 & 4 & 4 & 5 \\ 2 & -2 & -2 & -1 \end{bmatrix}$. The eigenvalues are $\lambda = 1, 2, 3$ Find the eigenvectors corresponding to each eigenvalue and determine if the matrix is diagonalizable. (20 points)

$\lambda = 1 \quad \begin{bmatrix} -6 & 10 & 4 & 14 \\ -7 & 10 & 5 & 13 \\ -3 & 4 & 3 & 5 \\ 2 & -2 & -2 & -2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $\quad x_1 = -x_4, \; x_2 = -2x_4, \; x_3 = 0, \; x_4 = x_4 \quad \vec{v}_1 = \begin{bmatrix} -1 \\ -2 \\ 0 \\ 1 \end{bmatrix}$

$\lambda = 2 \quad \begin{bmatrix} -7 & 10 & 4 & 14 \\ -7 & 9 & 5 & 13 \\ -3 & 4 & 2 & 5 \\ 2 & -2 & -2 & -3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $\quad x_1 = 2x_3, \; x_2 = x_3, \; x_3 = x_3, \; x_4 = 0 \quad \vec{v}_2 = \begin{bmatrix} 2 \\ 1 \\ 1 \\ 0 \end{bmatrix}$

$\lambda = 3 \quad \begin{bmatrix} -8 & 10 & 4 & 14 \\ -7 & 8 & 5 & 13 \\ -3 & 4 & 1 & 5 \\ 2 & -2 & -2 & -4 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & -3 & -3 \\ 0 & 1 & -2 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $\quad x_1 = 3x_3 + 3x_4, \; x_2 = 2x_3 + x_4, \; x_3 = x_3, \; x_4 = x_4 \quad \vec{v}_3 = \begin{bmatrix} 3 \\ 2 \\ 1 \\ 0 \end{bmatrix}, \vec{v}_4 = \begin{bmatrix} 3 \\ 1 \\ 0 \\ 1 \end{bmatrix}$

yes. the matrix is diagonalizable

5. Suppose that $\det(C) = 16$. Find the determinant of the matrix after the following row operations. (5 points)

$$6R_1 + R_2 \rightarrow R_2, R_4 \leftrightarrow R_3, 3R_2 + 4R_4 \rightarrow R_4, \frac{1}{4}R_4 \rightarrow R_4$$

$1 \qquad (-1) \qquad (4) \qquad (1/4)$

-16

6. Determine if the set formed by polynomials of the form $p(t) = a + bt + t^2$ is a subspace of P_n. If it is, prove it. If it is not, find an example to the contrary. (6 points each)

it is not a subspace not closed under addition

let $p(t) = a + bt + t^2 \quad g(t) = c + dt + t^2$

$p(t) + q(t) = (a+c) + (b+d)t + 2t^2$

coeff. is not 1 so not in set.

7. Consider the orthogonal basis for $\mathbb{R}^3$ given by $\left\{\begin{bmatrix}-1\\1\\3\end{bmatrix},\begin{bmatrix}0\\3\\-1\end{bmatrix},\begin{bmatrix}10\\1\\3\end{bmatrix}\right\}$. Use the property of Orthogonality to find the coordinate representation of the vector $\vec{x}=\begin{bmatrix}5\\7\\-1\end{bmatrix}$ in this basis. [Hint: no matrices are required.] (15 points)

$$c_1 = \frac{-5+7-3}{1+1+9} = \frac{-1}{11}$$

$$c_2 = \frac{0+21+1}{0+9+1} = \frac{22}{10} = \frac{11}{5}$$

$$c_3 = \frac{50+7+-3}{100+1+9} = \frac{54}{110} = \frac{27}{55}$$

$$[\vec{x}]_B = \begin{bmatrix}-1/11\\11/5\\27/55\end{bmatrix}$$

1. Given the vectors $\vec{u}=\begin{bmatrix}4\\-1\\2\\3\end{bmatrix}, \vec{v}=\begin{bmatrix}-2\\5\\3\\-4\end{bmatrix}$ find the following.

a. A unit vector in the direction of $\vec{v}$. (4 points)

$$\|\vec{v}\| = \sqrt{4+25+9+16} = \sqrt{54} = 3\sqrt{6}$$

$$\hat{v} = \begin{bmatrix}-2/3\sqrt{6}\\5/3\sqrt{6}\\1/\sqrt{6}\\-4/3\sqrt{6}\end{bmatrix}$$

b. Find the distance between $\vec{u}$ and $\vec{v}$. (7 points)

$$\vec{u}-\vec{v} = \begin{bmatrix}6\\-6\\-1\\7\end{bmatrix}$$

$$\|\vec{u}-\vec{v}\| = \sqrt{36+36+1+49} = \sqrt{122}$$

Determine if

8. ~~Show that~~ the polynomials $f(t) = 1 - 3t$, and $g(t) = 4 + 2t^2$ are orthogonal under the inner product $< f, g > = \int_{-1}^{1} f(t)g(t)dt$. (10 points)

$$\int_{-1}^{1}(1-3t)(4+2t^2)\,dt = \int_{-1}^{1} 4 \cancel{-12t} + 2t^2 \cancel{-6t^3}\,dt$$

odd terms

$$= 2\int_0^1 4 + 2t^2\,dt = 2\left[4t + \frac{2}{3}t^3\right|_0^1 = 2\left(4 + \frac{2}{3}\right) = \frac{28}{3}$$

they are not orthogonal

9. Determine if the following sets of vectors are linearly independent by inspection. Justify your answer in each case. (5 points each)

a. $\left\{\begin{bmatrix}1\\1\\1\end{bmatrix}, \begin{bmatrix}0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\1\end{bmatrix}\right\}$ not independent $\vec{0}$ in set

b. $\left\{\begin{bmatrix}1\\1\\1\\0\end{bmatrix}, \boxed{\begin{bmatrix}1\\2\\0\\3\end{bmatrix}, \begin{bmatrix}-2\\-4\\0\\-6\end{bmatrix}}, \begin{bmatrix}3\\5\\2\\1\end{bmatrix}\right\}$ $\vec{V_3} = -2\vec{V_2}$ not independent

c. $\left\{\begin{bmatrix}2\\3\end{bmatrix}, \begin{bmatrix}1\\1\end{bmatrix}\right\}$ they are independent

not multiples, only 2 vectors

10. Consider the stochastic Markov chain matrix given by the matrix $A = \begin{bmatrix} .8 & .02 \\ .3 & .98 \end{bmatrix}$. Calculate the equilibrium vector of the system. (5 points)

.2

$$\begin{bmatrix} -.2 & .02 \\ .2 & -.02 \end{bmatrix}$$

$\vec{X_1} = \frac{.02}{.2} X_2 \Rightarrow X_1 = .1X_2$

$X_2 = X_2$

$\vec{X} = \begin{bmatrix}1\\10\end{bmatrix}$

$$\vec{q} = \begin{bmatrix} 1/11 \\ 10/11 \end{bmatrix}$$

11. Suppose matrix A is a 9x5 matrix with 4 pivot columns. Determine the following. (10 points)

dim Col A = 4

dim Nul A = 1

dim Row A^T = 4

If Col A is a subspace of R^m, then m = 9

Rank A = 4

If Nul A is a subspace of R^n, then n = 5

12. Determine if each statement is True or False. (3 points each)

a. T F If vectors $\vec{v_1}, \vec{v_2}, \ldots \vec{v_p}$ span a subspace W and if $\vec{x}$ is orthogonal to each $\vec{v_j}$ for j=1...p, then $\vec{x}$ in $W^\perp$.

b. T F If $\vec{y}$ is in a subspace $W^\perp$, then the orthogonal projection of $\vec{y}$ onto W is $\vec{y}$ itself.
it $\vec{0}$

c. T F Every eigenvalue has only one corresponding eigenvector.

d. T F If A is a 7x5 matrix, then the transformation $\vec{x} \mapsto A\vec{x}$ can be onto but not one-to-one.
Cannot be onto

e. T F If a system of equations has a free variable then it has a unique solution.

f. T F The complex eigenvalues of a discrete dynamical system either both attract to the origin or both repel from the origin.

g. T F If two vectors are orthogonal, they are linearly independent. (assuming $\vec{v} \neq \vec{0}$, then true)

h. T F The third standard basis vector $\vec{e_4}$ in P_3 is t^3.

i. T F The null space of a matrix is a subspace of the codomain of the matrix.
of domain

j. T F If A is invertible, then A is diagonalizable.

k. T F A matrix is invertible if and only if 0 is not an eigenvalue of A.

l. T F If the columns of A are linearly independent, then the equation $A\vec{x} = \vec{b}$ has an infinite number least-squares solutions, or none at all.

m. T F A least-squares solution of $A\vec{x} = \vec{b}$ is the point in the column space of A closest to $\vec{b}$.

n. T F An isomorphism is a linear mapping from one n-dimensional space into another space of the same number of dimensions.

o. T F A matrix given by $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ has a unique solution if $ad - bc = 0$.

Name KEY

Math 2568, Final Exam – Part II, Fall 2014

Instructions: On this portion of the exam, you *may* use a calculator to perform elementary matrix operations. Support your answers with work (reproduce the reduced matrices from your calculator) or other justification for full credit.

1. Find a least squares solution for the set of points $\{(1,0.7),(2.1,3.4),(2.2,4.8),(3.1,11.7),(4.4,19.3),(5.3,25.8),(6.7,38.2)\}$ to satisfy the equation $y = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3$. Be sure to write the matrices employed, any equations, and the final regression function for y. (10 points)

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2.1 & 2.1^2 & 2.1^3 \\ 1 & 2.2 & 2.2^2 & 2.2^3 \\ 1 & 3.1 & 3.1^2 & 3.1^3 \\ 1 & 4.4 & 4.4^2 & 4.4^3 \\ 1 & 5.3 & 5.3^2 & 5.3^3 \\ 1 & 6.7 & 6.7^2 & 6.7^3 \end{bmatrix} \quad \vec{b} \begin{bmatrix} 0.7 \\ 3.4 \\ 4.8 \\ 11.7 \\ 19.3 \\ 25.8 \\ 38.2 \end{bmatrix} \quad (A^TA)^{-1}A^T\vec{b} = \vec{x} = \begin{bmatrix} -1.594 \\ .7671 \\ 1.166 \\ -.059 \end{bmatrix}$$

$$y = -1.594 + .7671x + 1.166x^2 - .059x^3$$

2. Given the vectors $\overrightarrow{b_1} = \begin{bmatrix} 3 \\ 1 \\ 8 \\ 2 \end{bmatrix}$ $and\ \overrightarrow{b_2} = \begin{bmatrix} 1 \\ -3 \\ 2 \\ -8 \end{bmatrix}$, find two more vectors orthogonal to these (and each other) to make an orthogonal basis for $\mathbb{R}^4$. (12 points)

$$\begin{bmatrix} 3 & 1 & 8 & 2 \\ 1 & -3 & 2 & -8 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 13/5 & -1/5 \\ 0 & 1 & 1/5 & 13/5 \end{bmatrix}$$

$$\begin{aligned} x_1 &= -13/5\, x_3 + 1/5\, x_4 \\ x_2 &= -1/5\, x_3 - 13/5\, x_4 \\ x_3 &= \quad x_3 \\ x_4 &= \quad\quad x_4 \end{aligned}$$

$$\vec{v}_1 = \begin{bmatrix} -13 \\ -1 \\ 5 \\ 0 \end{bmatrix} \quad \vec{v}_2 = \begin{bmatrix} 1 \\ -13 \\ 0 \\ 5 \end{bmatrix}$$

3. The set $H = \left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix} \right\}$ forms a basis for R^3 . Use the Gram-Schmidt Process to make an orthogonal basis, and then normalize it. (15 points)

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

$$\vec{v}_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} - \frac{0+0-1}{1+1+1}\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + \frac{1}{3}\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1/3 \\ 1/3 \\ 2/3 \end{bmatrix} \to \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$$\vec{v}_3 = \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix} - \frac{1+2+2}{1+1+1}\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} - \frac{1+2-4}{1+1+4}\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix} - \frac{5}{3}\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} + \frac{1}{6}\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} -1/2 \\ 1/2 \\ 0 \end{bmatrix} \to \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

Orthogonal basis

$$\left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \right\}$$

orthonormal basis

$$\left\{ \begin{bmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ -1/\sqrt{3} \end{bmatrix}, \begin{bmatrix} 1/\sqrt{6} \\ 1/\sqrt{6} \\ 2/\sqrt{6} \end{bmatrix}, \begin{bmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{bmatrix} \right\}$$

4. Given the basis of $W = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 2 \end{bmatrix} \right\}$, and the vector $\vec{y} = \begin{bmatrix} 7 \\ 2 \\ -3 \\ 0 \end{bmatrix}$ decompose this vector into $\vec{y} = \vec{y_\parallel} + \vec{y_\perp}$ with $\vec{y_\parallel} = proj_W\vec{y}$. (10 points)

$$Y_{\parallel} = \frac{7+2+0+0}{1+1+0+0}\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \frac{9}{2}\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 9/2 \\ 9/2 \\ 0 \\ 0 \end{bmatrix}$$

$$+ \frac{0+0-3+0}{1+4+0+0}\begin{bmatrix} 0 \\ 0 \\ 1 \\ 2 \end{bmatrix} = \frac{-3}{5}\begin{bmatrix} 0 \\ 0 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -3/5 \\ -6/5 \end{bmatrix}$$

$$= \begin{bmatrix} 9/2 \\ 9/2 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -3/5 \\ -6/5 \end{bmatrix} = \begin{bmatrix} 9/2 \\ 9/2 \\ -3/5 \\ -6/5 \end{bmatrix} = \vec{Y}_{\parallel}$$

$$\vec{Y}_{\perp} = \begin{bmatrix} 7 \\ 2 \\ -3 \\ 0 \end{bmatrix} - \begin{bmatrix} 9/2 \\ 9/2 \\ -3/5 \\ -6/5 \end{bmatrix} = \begin{bmatrix} 5/2 \\ -5/2 \\ -12/5 \\ 6/5 \end{bmatrix}$$

$$\vec{Y} - \vec{Y}_{\parallel}$$

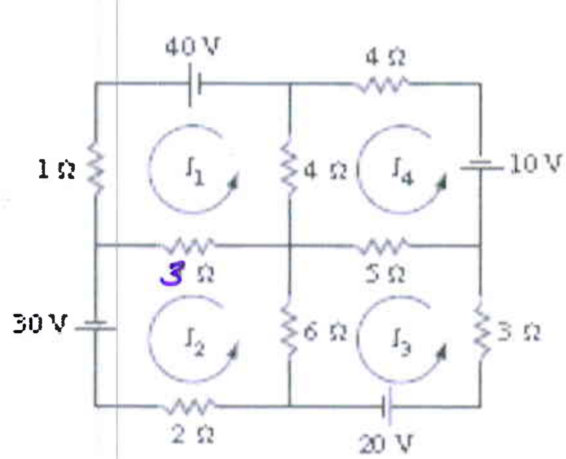
5. Assume that $A=\begin{bmatrix} 1 & 3 & 0 & 5 & 0 & 3 \\ 2 & 2 & -1 & 2 & 2 & -5 \\ 1 & -1 & 3 & -3 & 1 & 9 \\ 5 & 4 & 1 & 3 & 1 & 7 \end{bmatrix}$. Find a basis for the null space of *A*. (8 points)

$$\xrightarrow{\text{rref}} \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 5/19 \\ 0 & 1 & 0 & 2 & 0 & 52/57 \\ 0 & 0 & 1 & 0 & 0 & 217/57 \\ 0 & 0 & 0 & 0 & 1 & -101/57 \end{bmatrix}$$

$$\begin{aligned} x_1 &= x_4 - 5/19\, x_6 \\ x_2 &= -2x_4 - 52/57\, x_6 \\ x_3 &= -217/57\, x_6 \\ x_4 &= x_4 \\ x_5 &= +101/57\, x_6 \\ x_6 &= x_6 \end{aligned}$$

$$\text{Nul } A = \left\{ \begin{bmatrix} 1 \\ -2 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -15 \\ -52 \\ -217 \\ 0 \\ 101 \\ 57 \end{bmatrix} \right\}$$

6. Based on the graph below, solve the system for the circuit flowing through each loop. You may round your answers to three decimal places as needed. (10 points)



$$8I_1 - 3I_2 - 4I_4 = 40$$

$$-3I_1 + 11I_2 - 6I_3 = 30$$

$$-6I_2 + 14I_3 - 5I_4 = 20$$

$$-4I_1 - 5I_3 + 13I_4 = -10$$

$$\begin{bmatrix} 8 & -3 & 0 & -4 \\ -3 & 11 & -6 & 0 \\ 0 & -6 & 14 & -5 \\ -4 & 0 & -5 & 13 \end{bmatrix} \vec{x} = \begin{bmatrix} 40 \\ 30 \\ 20 \\ -10 \end{bmatrix}$$

$$\vec{I} = \begin{bmatrix} 11.824 \\ 10.293 \\ 7.957 \\ 5.930 \end{bmatrix}$$

7. The following are short answer questions. Always provide justification for any answers. You may use examples as part of your explanations, but if you are asked to "explain" your answer must contain **words**. (4 points each)

   a. Give an example of a 5x5 matrix with a non-trivial solution.

$$\begin{bmatrix} 1 & 0 & 1 & 3 & 0 \\ 0 & 1 & 1 & 4 & 0 \\ 0 & 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

pivot in every column
$\Rightarrow$ trivial.
this one has no pivot
in Col#5

   b. Why must an nxn matrix have n ***distinct*** eigenvalues to guarantee that the eigenspace spans $\mathbb{R}^n$?

repeated eigenvalues are not
guaranteed to have multiple vectors
however each eigenvalue must have at
least 1 vectors so n roots $\Rightarrow$ n vectors.

   c. Give two properties of the invertible matrix theorem and explain why they must be equivalent to each other.

answers will vary.

A is invertible when A has all non zero eigenvalues
and since A^T has the same determinant as A,
it must also have the same eigenvalues so
A^T is invertible. The reverse is also true.

   d. Give an example of a stochastic matrix that has more than one equilibrium vector.

$$\begin{bmatrix} .9 & .8 & 0 & 0 \\ .1 & .2 & 0 & 0 \\ 0 & 0 & .3 & .6 \\ 0 & 0 & .7 & .4 \end{bmatrix}$$

e. Explain why the equation y=mx+b is not a linear transformation under the definitions used in this course.

Since

$y_1 + y_2 = mx_1 + mx_2 + 2b$

but needs to be $mx_1 + mx_2 + b$

to satisfy $m(x_1+x_2)+b$

not closed under addition

f. Explain why the complex eigenvalues of a discrete dynamical system cannot produce a saddle point.

Since both eigenvalues are

$\lambda = a \pm bi$ and $|\lambda| = a^2+b^2$ for both

So it cannot be the case that

a^2+b^2 is <1 and the other a^2+b^2 is >1.

which is needed for a saddle point.

g. What are the advantages and disadvantages of finding determinants by row-reducing compared to the cofactor method?

Cofactor works best where there are a lot of zeros or if matrix is relatively small (say 3x3).

row-reducing has fewer operations for very large matrices.

h. Give at least two reasons why being able to diagonalize a matrix is so important computationally.

diagonal matrices can be raised to powers easily, made exponents of e and other algebra is simpler. When no calculator is available, it significantly shortens computations.

i. Explain the relationship between a vector $\vec{y}$ in $\mathbb{R}^n$, W a subspace of $\mathbb{R}^n$, $\vec{v}, \vec{y_{\|}}$ which are vectors in W, as described by the Best Approximation Theorem.

The distance $\|\vec{y} - \vec{y}_{\|}\|$ in W is less than the distance $\|\vec{y} - \vec{v}\|$ for any $\vec{v} \neq \vec{y}_{\|}$ in W.