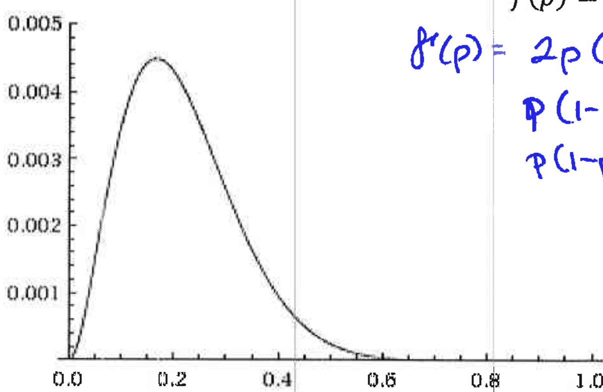
Stat 2470, In-Class Exercise #1, Spring 2014
Maximum Likelihood Functions

$$f(p) = p^2(1-p)^{10}$$



$f'(p) = 2p(1-p)^{10} - 10(1-p)^9 p^2 = 0$

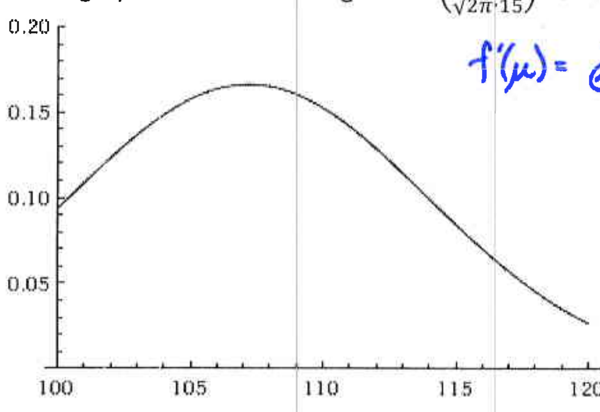
$p(1-p)^9[2(1-p) - 10p] = 0$

$p(1-p)^9[2 - 12p] = 0$ $\quad p = 0, 1$ and $1/6 = p$

min $\quad$ max

$$f(\mu) = e^{-\frac{(110-\mu)^2}{2\cdot 15^2}} e^{-\frac{(98-\mu)^2}{2\cdot 15^2}} e^{-\frac{(88-\mu)^2}{2\cdot 15^2}} e^{-\frac{(123-\mu)^2}{2\cdot 15^2}} e^{-\frac{(117-\mu)^2}{2\cdot 15^2}} = e^{-\frac{1}{2\cdot 15^2}\left(\sum (x_i-\mu)^2\right)}$$

*This graph omits the scaling factor $\left(\frac{1}{\sqrt{2\pi}\cdot 15}\right)^5$ since it does not affect the location of the peak.

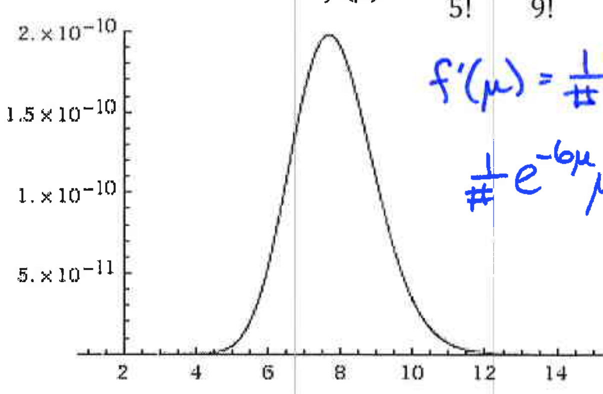


$f'(\mu) = e^{-\frac{1}{2\cdot 15^2}\left(\sum (x_i-\mu)^2\right)} \cdot \left(-\frac{1}{2\cdot 15^2} \sum (x_i-\mu)\, 2\cdot(-1)\right) = 0$

$\sum (x_i - \mu) = 0 \Rightarrow \sum x_i - \sum \mu = 0$

$\sum x_i = \sum \mu = 5\mu \Rightarrow \mu = \frac{\sum x_i}{5} = \frac{536}{5} = 107.2$

$$f(\mu) = \frac{e^{-\mu}\mu^5}{5!}\frac{e^{-\mu}\mu^9}{9!}\frac{e^{-\mu}\mu^{12}}{12!}\frac{e^{-\mu}\mu^3}{3!}\frac{e^{-\mu}\mu^{16}}{16!}\frac{e^{-\mu}\mu^1}{1!} = \frac{e^{-6\mu}\mu^{46}}{\text{some large \#}}$$



$f'(\mu) = \frac{1}{\#}\left[-6e^{-6\mu}\mu^{46} + e^{-6\mu}46\mu^{45}\right] = 0$

$\frac{1}{\#}e^{-6\mu}\mu^{45}[-6\mu + 46] = 0 \Rightarrow 46 = 6\mu \Rightarrow \mu = \frac{46}{6} = \frac{23}{3}$

1. Suppose that you took 7 samples of a Bernoulli random variable and obtained 5 successes. Write the equation of the maximum likelihood estimator. Then find the MLE estimate for $\hat{p}$.

$$p^5(1-p)^2 = L(p)$$

$$L'(p) = 5p^4(1-p)^2 - 2p^5(1-p) = 0$$

$$p^4(1-p)[5(1-p) - 2p] = [5-7p]p^4(1-p) = 0$$

$$5-7p=0$$

$$p = \frac{5}{7}$$

max

$p=0, p=1$ are both minima

2. Suppose that the number of wild rabbits that investigate a no-kill trap is distributed as a Poisson random variable. Hiding in a blind, an ecologist captures and tags 2, 6, 3, 8, 0, 1 animals in the next six hours. Write the equation of the maximum likelihood estimator. Then find the MLE estimate for $\hat{\mu}$.

$$L(\mu) = \frac{e^{-\mu}\mu^2}{2!} \frac{e^{-\mu}\mu^6}{6!} \cdot \frac{e^{-\mu}\mu^3}{3!} \cdot \frac{e^{-\mu}\mu^8}{8!} \cdot \frac{e^{-\mu}\mu^0}{0!} \cdot \frac{e^{-\mu}\mu^1}{1!} =$$

$$\frac{1}{\text{large \#}} e^{-6\mu}\mu^{20}$$

$$L'(\mu) = \frac{1}{\#}\left[-6e^{-6\mu}\mu^{20} + 20\mu^{19}e^{-6\mu}\right] = 0$$

$$\frac{1}{\#} e^{-6\mu}\mu^{19}[-6\mu + 20] = 0$$

$\mu = 0$ min

$$20 = 6\mu$$

$$\mu = \frac{20}{6} = \frac{10}{3}$$

max.

3. Suppose that you measure the heights of 4 women (heights are distributed normally) and obtain the results (in cm): 155, 170, 152, 160. Write the equation for the maximum likelihood function and use it to find the MLE estimates for $\hat{\mu}$ and $\hat{\sigma}$.

$$L(\mu,\sigma) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(155-\mu)^2}{2\sigma^2}} \cdot \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(170-\mu)^2}{2\sigma^2}} \cdot \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(152-\mu)^2}{2\sigma^2}} \cdot \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(160-\mu)^2}{2\sigma^2}}$$

$$L(\mu,\sigma) = \frac{1}{(2\pi)^2\sigma^4} e^{-\frac{1}{2\sigma^2}\left[(155-\mu)^2+(170-\mu)^2+(152-\mu)^2+(160-\mu)^2\right]}$$

$$\frac{\partial L}{\partial \mu} = \frac{1}{(2\pi)^2\sigma^4} e^{-\frac{1}{2\sigma^2}\left[\Sigma (x_i-\mu)^2\right]} \left[\Sigma (x_i-\mu)\cdot 2(-1)\left(\frac{-1}{2\sigma^2}\right)\right] = 0 \Rightarrow \Sigma(x_i-\mu)=0$$

$$\mu = \frac{\Sigma x_i}{4} = \frac{637}{4} = 159.25$$

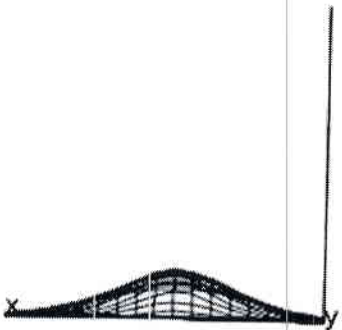
$$\frac{\partial L}{\partial \sigma} = \frac{1}{(2\pi)^2}(-4\sigma^{-5}) e^{-\frac{1}{2\sigma^2}\Sigma(x_i-\mu)^2} + \frac{1}{(2\pi)^2\sigma^4} e^{-\frac{1}{2\sigma^2}\Sigma(x_i-\mu)^2} \cdot \left(\tfrac{-1}{2}(-2)(\sigma^{-3})\right)\Sigma(x_i-\mu)^2$$

$$= \frac{1}{(2\pi)^2\sigma^7} e^{-\frac{1}{2\sigma^2}\Sigma(x_i-\mu)^2}\left[-4\sigma^2 + \Sigma(x_i-\mu)^2\right] = 0$$

$$\sigma^2 = \frac{\Sigma(x_i-\mu)^2}{4} = \frac{(155-159.25)^2+(170-159.25)^2+(152-159.25)^2+(160-159.25)^2}{4}$$

$$\sigma^2 = 46.6875$$

$$\sigma = \sqrt{46.6875} \approx 6.832825$$



In these graphs x is μ and y is σ.
x is plotted between 150 and 170.
y is plotted between 2 and 10. Do these graphs agree with your estimates from the math?