

Instructions: Show all work. Answers without work may only receive partial credit. If you are asked for an explanation, explain as completely as possible. Use exact answers unless specifically asked to round.

1. Find the derivative of the following functions.

a. $y = e^{\sqrt{x}} = e^{x^{1/2}}$

$$e^{\sqrt{x}} \cdot \frac{1}{2} \frac{1}{\sqrt{x}} = \boxed{\frac{e^{\sqrt{x}}}{2\sqrt{x}}}$$

b. $y = \sin(4x^3 + 3x + 1)$

$$\cos(4x^3 + 3x + 1) (12x^2 + 3) = \boxed{(12x^2 + 3) \cos(4x^3 + 3x + 1)}$$

c. $y = \sin^5(\cos x^2)$

$$5 \sin^4(\cos x^2) \cdot \cos(\cos x^2) \cdot (-\sin x^2) \cdot 2x$$

$$= \boxed{-10x \sin^4(\cos x^2) \cos(\cos x^2) \sin x^2}$$

d. $y = \tan(xe^x)$

$$\sec^2(xe^x)(e^x + xe^x) = \boxed{e^x(1+x) \sec^2(xe^x)}$$

e. $y = \sec^{-1}(\ln x) = \arccos(\ln x)$

$$\frac{1}{|\ln x| \sqrt{(\ln x)^2 - 1}} \cdot \frac{1}{x} = \boxed{\frac{1}{x |\ln x| \sqrt{(\ln x)^2 - 1}}}$$

f. $y = \sin^{-1}(2x)$

$$\frac{1}{\sqrt{1-4x^2}} \cdot 2 = \boxed{\frac{2}{\sqrt{1-4x^2}}}$$

2. Find the derivative $\frac{dy}{dx}$ of the implicit function $xy^3 + 1 = x - y^2 + \cos(y)$.

$$y^3 + 3xy^2 \cdot y' = 1 - 2yy' - \sin(y)y'$$

$$y^3 - 1 = -3xy^2y' - 2yy' - \sin(y)y'$$

$$1 - y^3 = y'(3xy^2 + 2y + \sin y)$$

$$\boxed{\frac{1 - y^3}{3xy^2 + 2y + \sin y} = y' = \frac{dy}{dx}}$$

3. Use logarithmic differentiation to find the derivative of $y = x^{\ln x}$.

$$\ln y = \ln(x^{\ln x})$$

$$\ln y = \ln x \cdot \ln x = \ln^2 x$$

$$\frac{1}{y} y' = 2 \ln x \cdot \frac{1}{x}$$

$$y' = \frac{2 \ln x}{x} \cdot y$$

$$\boxed{y' = \frac{2 \ln x}{x} \cdot x^{\ln x}}$$