

Instructions: Show all work. Answers without work may only receive partial credit. If you are asked for an explanation, explain as completely as possible. Use exact answers unless specifically asked to round.

1. Find the derivatives of the following functions.

a. $y = \sec(x) + \csc(x)$

$$y' = \sec x \tan x - \csc x \cot x$$

b. $y = e^{5x} \tan(x)$

$$y' = 5e^{5x} \tan x + e^{5x} \sec^2 x$$

c. $y = \frac{1 - \cos(x)}{1 + \cos(x)}$

$$y' = \frac{\sin x (1 + \cos x) + \sin x (1 - \cos x)}{(1 + \cos x)^2} = \frac{2 \sin x}{(1 + \cos x)^2}$$

2. A stone is thrown from the edge of a bridge that is 48 ft above the ground with an initial velocity of 32 ft/s. The height of this stone above the ground t seconds after it is thrown is $f(t) = -16t^2 + 32t + 48$. If a second stone is thrown from the ground then its height above the ground after t seconds is $g(t) = -16t^2 + v_0 t$ where v_0 is the initial velocity of the second stone. Determine the value of v_0 so that both stones reach the same high point. [Hint: start by finding out how high the first stone gets.]

$$f'(t) = -32t + 32 = 0 \Rightarrow t = 1$$

$$f(1) = -16(1) + 32(1) + 48 = 64 \text{ ft. high}$$

$$g(t) = -16t^2 + v_0 t = 64 \Rightarrow -16 \left(\frac{v_0}{32}\right)^2 + v_0 \left(\frac{v_0}{32}\right) = 64 \Rightarrow \frac{v_0^2}{-64} + \frac{v_0^2}{32} = 64$$

$$g'(t) = -32t + v_0 = 0 \Rightarrow v_0 = 32t$$

$$t = \frac{v_0}{32}$$

$$\frac{+v_0^2}{64} = 64$$

$$\frac{v_0^2}{32} = 64^2$$