

Instructions: Show all work. Answers without work may only receive partial credit. If you are asked for an explanation, explain as completely as possible. Use exact answers unless specifically asked to round.

1. Find the derivative and the second derivative of the following functions.

a. $f(x) = 10x^4 - 32x + e^2$

$$f'(x) = 40x^3 - 32$$

$$f''(x) = 120x^2$$

b. $g(w) = \frac{w^3 - w}{w} = w^2 - 1$

$$g'(w) = 2w$$

$$g''(w) = 2$$

2. Find the equation of the tangent line at $y = \frac{e^x}{4} - x$, when $x = 0$.

$$y' = \frac{1}{4}e^x - 1 \quad y'(0) = \frac{1}{4}e^0 - 1 = \frac{1}{4}(1) - 1 = -\frac{3}{4} \text{ slope}$$

$$y(0) = \frac{e^0}{4} - 0 = \frac{1}{4} \quad \text{point on graph } (0, \frac{1}{4})$$

$$y - y_1 = m(x - x_1) \Rightarrow y - \frac{1}{4} = -\frac{3}{4}(x - 0) \Rightarrow \boxed{y = -\frac{3}{4}x + \frac{1}{4}}$$

3. Use the product rule and the quotient rule as appropriate to find the derivative of the functions.

a. $h(x) = \frac{e^x}{e^x + 1}$

$$f = e^x \quad g = e^x + 1$$

$$f' = e^x \quad g' = e^x$$

$$h'(x) = \frac{e^x(e^x + 1) - e^x \cdot e^x}{(e^x + 1)^2} = \frac{e^x}{(e^x + 1)^2}$$

b. $m(x) = (x^2 - 1)e^{-x}$

$$f = x^2 - 1 \quad g = e^{-x}$$

$$f' = 2x \quad g' = -e^{-x}$$

$$-(x^2 - 1)e^{-x} + 2xe^{-x}$$

$$\text{or } \boxed{2xe^{-x} - (x^2 - 1)e^{-x}}$$