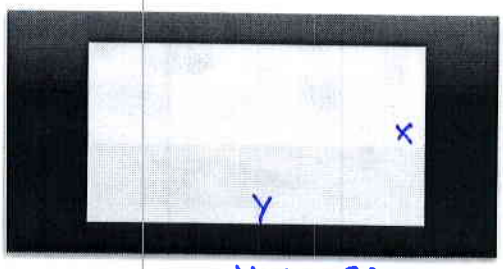
Math 1151, Quiz #10, Fall 2014 Name KEY

Instructions: Show all work. Answers without work may only receive partial credit. If you are asked for an explanation, explain as completely as possible. Use exact answers unless specifically asked to round.

1. A rectangular garden with an area of 30 m^2 is surrounded by a grass border 1 m wide on two sides and 2 m wide on the other two sides. What dimensions of the garden minimize the combined area of the garden and borders?



$A = \ell w = yx = 30$

$y = \frac{30}{x}$

$x+2$ $A = (x+2)\left(\frac{30}{x}+4\right)$

$= 30 + 4x + \frac{60}{x} + 8 = 38 + 4x + \frac{60}{x}$

$y+4 = \frac{30}{x}+4$

$A' = 4 - \frac{60}{x^2} = 0$ $4x^2 = 60$

$x = \pm\sqrt{15}$ neg. $-\sqrt{15}$

$x = \sqrt{15}$

$y = \frac{30}{\sqrt{15}} = 2\sqrt{15}$

2. For the function $f(x) = 12 - x^2, a = 2$ use the differential to approximate the value of $x = 2.1$ and use the approximation and the true value $f(2.1)$ to calculate the percent error.

$dy = (-2x)dx$

$dy = -2(2)\,dx = -4\,dx$

$dx = 2.1 - 2 = .1$

$f(2) = 12 - 4 = 8$

$y \approx dy + y(2)$

$y \approx -4(.1) + 8 = 7.6$

$f(2.1) = 12 - (2.1)^2 = 7.59$

$\frac{7.6 - 7.59}{7.59} = .001375$

$\approx .1\%$

3. Determine whether the Mean Value Theorem applies to the function $f(x) = 2\sqrt[3]{x}$ on the interval [-8,8].

Continuous

$f(-8) = 2(-2) = -4$

$f(8) = 2(2) = 4$

$f'(x) = \frac{2}{3}x^{-2/3} = \frac{2}{3\sqrt[3]{x^2}}$

$\frac{4-(-4)}{8-(-8)} = \frac{8}{16} = \frac{1}{2}$

$\frac{2}{3\sqrt[3]{x^2}} = \frac{1}{2} \Rightarrow 3\sqrt[3]{x^2} = 4$

$\sqrt[3]{x^2} = \frac{4}{3}$

$x^2 = \frac{64}{27}$

$x = \pm\frac{8}{3\sqrt{3}}$

it does. on $(-8,0)$ and $(0,8)$ but not $(-8,8)$