Math 1151, Mathematical Induction Activity, Fall 2014 Name KEY

Instructions: Use the proof of $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ provided in class as a model, to prove the following summation formulas with the same technique.

1. Prove $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

base case: $n=1$ $\sum_{i=1}^{1} i^2 = 1^2 = 1$ and $\frac{1(1+1)(2(1)+1)}{6} = \frac{2(3)}{6} = 1$

induction case. Suppose true for k, prove for k+1

$\sum_{i=1}^{k} i^2 = \frac{k(k+1)(2k+1)}{6}$ then $\sum_{i=1}^{k+1} i^2 = \sum_{i=1}^{k} i^2 + (k+1)^2 =$

$\frac{k(k+1)(2k+1)}{6} + (k+1)^2 = (k+1)\left[\frac{k(2k+1)}{6} + (k+1)\frac{6}{6}\right] = (k+1)\left[\frac{2k^2+k+6k+6}{6}\right]$

$= (k+1)\left[\frac{2k^2+7k+6}{6}\right] = \left(\frac{k+1}{6}\right)\left[(2k+3)(k+2)\right] = \frac{(k+1)(k+2)(2k+3)}{6}$

$= \frac{(k+1)(k+1+1)(2(k+1)+1)}{6}$ which was to be shown so the formula works for all n

2. Prove $\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$

base case: $n=1$ $\sum_{i=1}^{1} i^3 = 1^3 = 1$ and $\frac{1^2(1+1)^2}{4} = \frac{2^2}{4} = 1$ it works

induction case: Suppose true for k, and prove for k+1.

$\sum_{i=1}^{k} i^3 = \frac{k^2(k+1)^2}{4}$ then $\sum_{i=1}^{k+1} i^3 = \sum_{i=1}^{k} i^3 + (k+1)^3 = \frac{k^2(k+1)^2}{4} + (k+1)^3$

$= (k+1)^2\left[\frac{k^2}{4} + (k+1)\frac{4}{4}\right] = \frac{(k+1)^2}{4}\left[k^2+4k+4\right] = \frac{(k+1)^2(k+2)^2}{4} =$

$\frac{(k+1)^2(k+1+1)^2}{4}$ which is what was to be shown. So the formula works for all n.