

Instructions: Find y' for each of the implicit functions below.

1. $\sqrt{x} + \sqrt{y} = 16$

$$\begin{aligned} x^{1/2} + y^{1/2} &= 16 \\ \left(\frac{1}{2}x^{-1/2} + \frac{1}{2}y^{-1/2}y' \right) 2\sqrt{y} &= 0 \\ -\frac{\sqrt{y}}{\sqrt{x}} &= y' \end{aligned}$$

2. $x^5 - xy + y^2 = 7$

$$\begin{aligned} 5x^4 - y - xy' + 2yy' &= 0 \\ y'(-x + 2y) &= y - 5x^4 \\ y' &= \frac{y - 5x^4}{2y - x} \end{aligned}$$

3. $x^2y + y^2x = -3$

$$\begin{aligned} 2xy + x^2y' + 2yy' + y^2 &= 0 \\ 2xy + y^2 &= -y'(x^2 + 2yx) \\ y' &= -\frac{(2xy + y^2)}{x^2 + 2yx} \end{aligned}$$

4. $e^{xy}y^3 - y = x$

$$\begin{aligned} e^{xy}(y + xy')y^3 + e^{xy}3y^2y' - y' &= 1 \\ ye^{xy} + xy^3e^{xy}y' + 3y^2e^{xy}y' - y' &= 1 - y^4e^{xy} \\ y' &= \frac{1 - y^4e^{xy}}{xy^3e^{xy} + 3y^2e^{xy} - 1} \end{aligned}$$

5. $\sin(x) = x(1 - \tan(y + x))$

$$\cos x = 1 - \tan(y + x) + x \sec^2(y + x)(y' + 1)$$

$$\cos x - 1 + \tan(y + x) - x \sec^2(y + x) = x \sec^2(y + x)y'$$

$$y' = \frac{\cos x - 1 + \tan(y + x) - x \sec^2(y + x)}{x \sec^2(y + x)}$$

6. $\ln(xy) + 5x = 30$

$$\ln x + \ln y + 5x = 30$$

$$\frac{1}{x} + \frac{1}{y}y' + 5 = 0$$

$$y' = \left(-\frac{1}{x} - 5\right)y$$

$$y' = -\frac{1}{x} - 5y$$