Math 1151, Definition of a Limit Activity, Fall 2014 Name KEY

Instructions: The definition of a limit is shown below. Use this definition to prove the limit for each of the functions below by finding the relationship between ϵ and δ, and laying out the proof in the correct format.

Definition of a Limit

Let f be a function defined on an open interval containing c (except possibly at c), and let L be a real number. The statement $\lim_{x \to c} f(x) = L$ means that for each $\epsilon > 0$ there exists a $\delta > 0$ such that if $0 < |x - c| < \delta$, then $|f(x) - L| < \epsilon$.

Steps

1) Find the proposed limit value algebraically.
2) Set $|f(x) - L| < \epsilon$ and try to rewrite the expression in terms of $|x - c|$, and use that relationship to find δ.
3) Prove the relationship satisfies the definition by beginning the proof with $|x - c| < \delta$ and use the information obtained in step #2 to find $|f(x) - L|$ in terms of ϵ.

1. $\lim_{x \to 2}(3x + 2)$

Suppose that $|x-2| < \delta$ and let $\delta = \varepsilon/3$, then $|x-2| < \varepsilon/3$ $\Rightarrow$ $3|x-2| < \varepsilon$ $\Rightarrow$ $|3x-2| = |(3x+2)-8| < \varepsilon$ $\therefore$ $\lim_{x \to 2}(3x+2) = 8$.

2. $\lim_{x \to 4}\left(4 - \frac{x}{2}\right)$

Suppose that $|x-4| < \delta$ and let $\delta = 2\varepsilon$. Then $|x-4| < 2\varepsilon$ $\Rightarrow$ $\frac{1}{2}|x-4| = \frac{1}{2}|4-x| < \varepsilon$ $\Rightarrow$ $|2 - x/2| = |(4-\frac{x}{2})-2| < \varepsilon$ $\therefore$ $\lim_{x \to 4}(4 - \frac{x}{2}) = 2$

3. $\lim_{x \to 2}(x^2 - 3)$

Suppose that $|x-2| < \delta$ and let $\delta = \varepsilon/5$. then $|x-2| < \varepsilon/5$ $\Rightarrow$ $5|x-2| < \varepsilon$ $|x+2| \le 5$ near $x=2$ (specifically on $[1,3]$) Therefore $|x+2||x-2| < \varepsilon$ $\Rightarrow$ $|x^2-4| < \varepsilon$ $\Rightarrow$ $|(x^2-3)-1| < \varepsilon$. $\therefore$ $\lim_{x \to 2}(x^2-3) = 1$

4. $\lim_{x \to -3}(2x + 5)$

Suppose that $|x-(-3)| = |x+3| < \delta$. and let $\delta = \varepsilon/2$. Then $|x+3| < \varepsilon/2$ $\Rightarrow$ $2|x+3| < \varepsilon$ $\Rightarrow$ $|2x+6| = |(2x+5)+1| =$ $|(2x+5)-(-1)| < \varepsilon$. $\therefore$ $\lim_{x \to -3}(2x+5) = -1$.