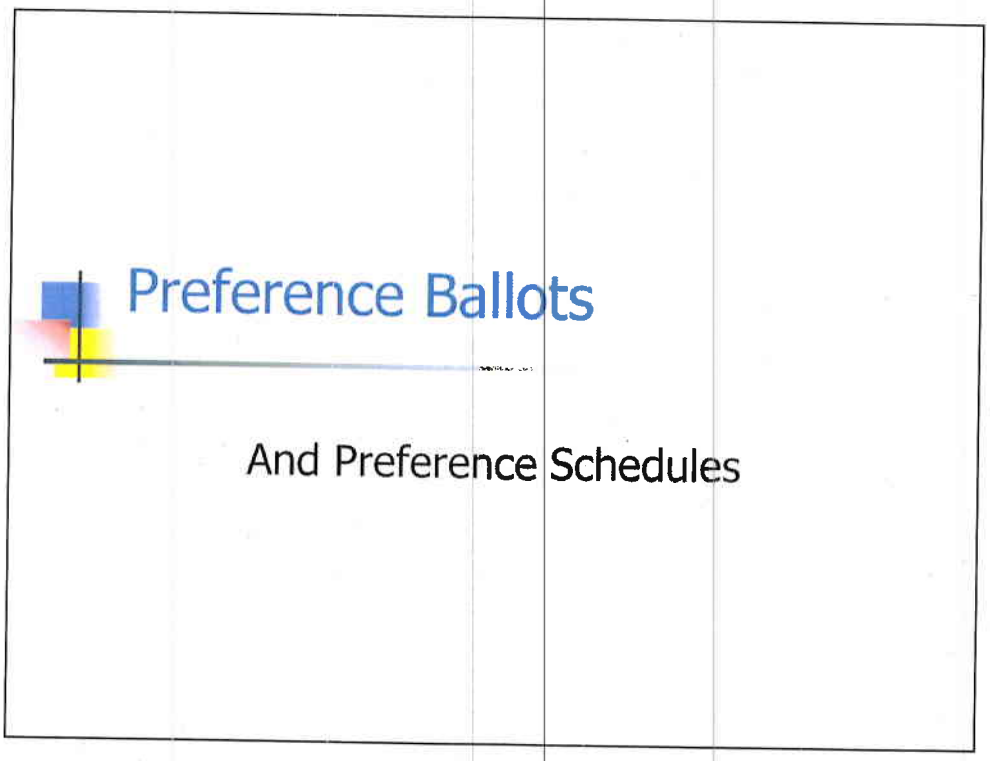
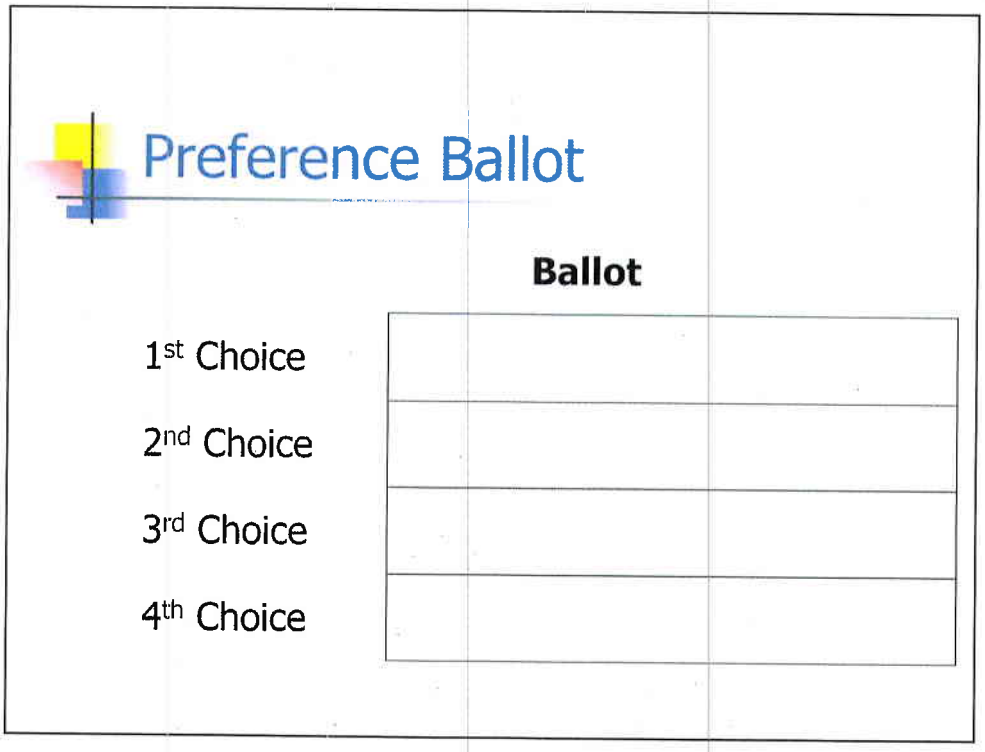
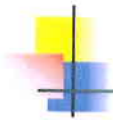

Preference Ballots
And Preference Schedules



Rather than just vote for "Cuthbert," if there are 4 candidates, list all 4 in order of preference. That's a preference ballot.



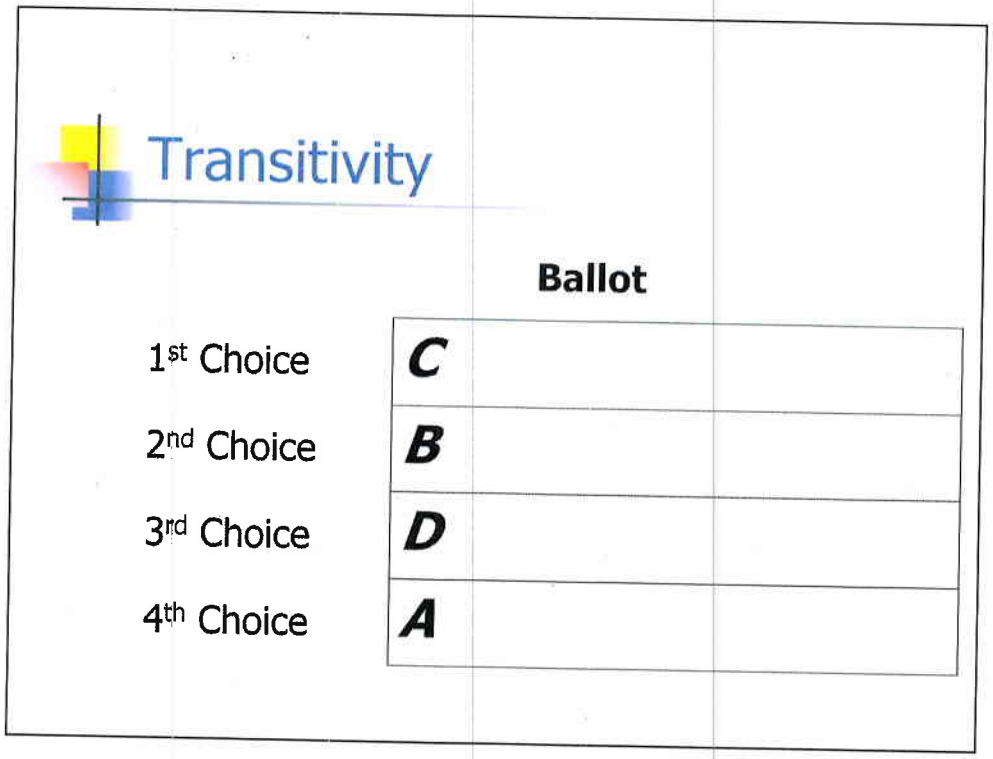
Preference Ballot

	Ballot
1st Choice	*Cuthbert*
2nd Choice	*Adolf*
3rd Choice	*Delbert*
4th Choice	*Bertha*

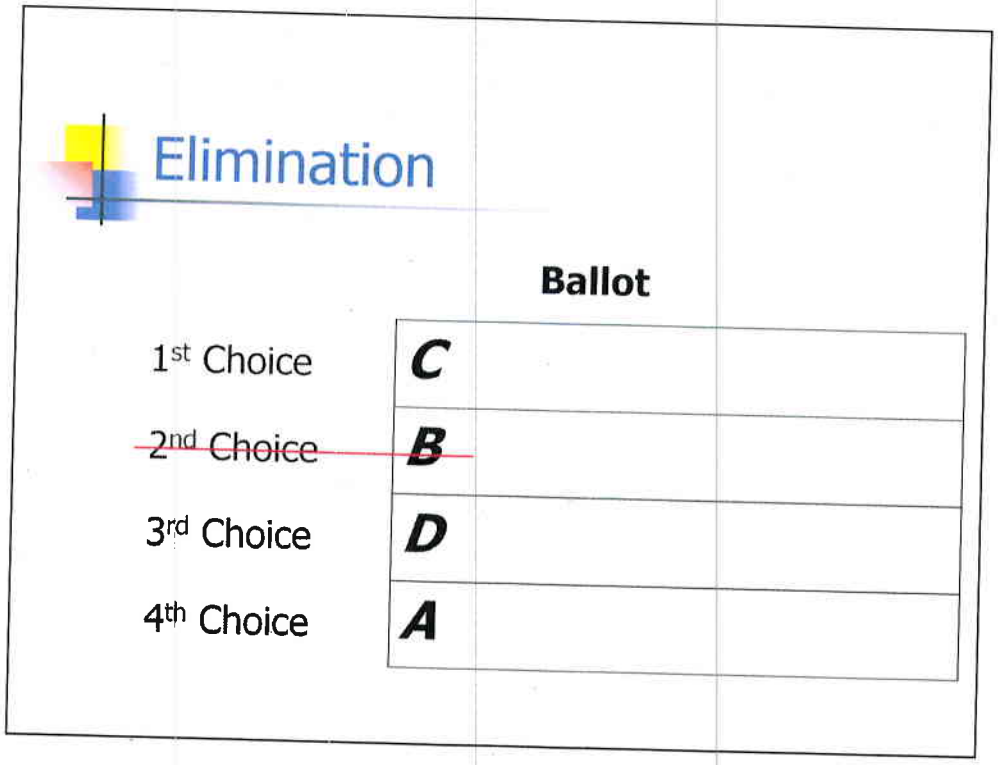
Preference Schedule

Voters	14	10	8	4	1
1st choice	A	C	D	B	C
2nd choice	B	B	C	D	D
3rd choice	C	D	B	C	B
4th choice	D	A	A	A	A

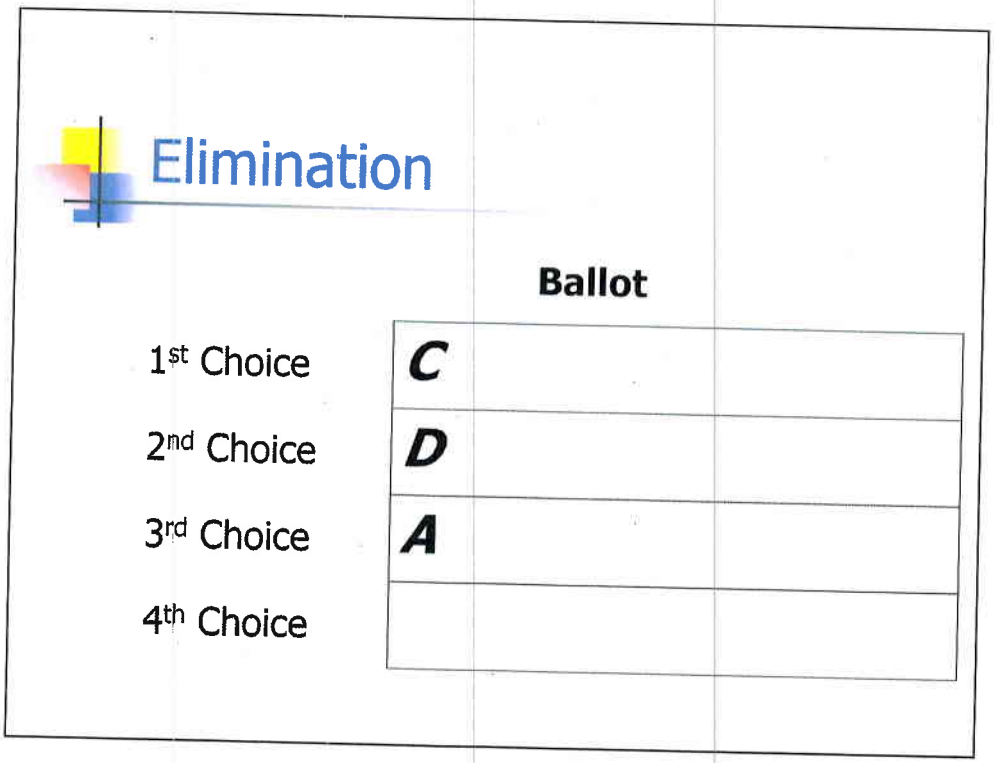
We then group the ballots (there are only 24 different patterns – Why?) and tally them up, giving what is known as a preference schedule.



Transitivity means that if C is over B and B is over D, then C is over D.



This means, for example, if Bertha drops out…



…we can simply erase Bertha and move those below up a space, without changing anyone's preferences.

Now, how do we decide the results?

There are a number of methods. The first is the "plurality method."

Plurality Method

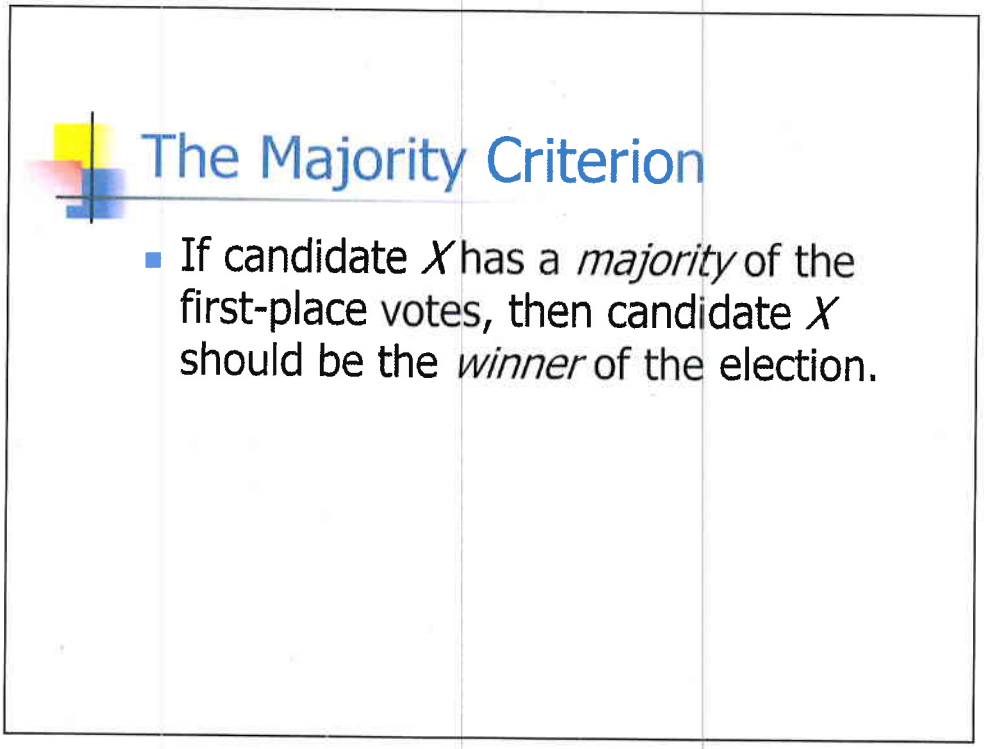
Voters	14	10	8	4	1
1st choice	A	C	D	B	C
2nd choice	B	B	C	D	D
3rd choice	C	D	B	C	B
4th choice	D	A	A	A	A

--the one with the most first choice votes wins.

--this is the method used in most United States elections (we just don't write down our 2nd, 3rt, etc. choices)

--natural extension of "Majority Rule"

--the problem: if there are many candidates, it is possible to be elected with only a small percentage of the votes.



This is a fundamental principle of a democratic election.

What does *majority* mean? More than half.

Notice that the plurality method satisfies this criterion: if a candidate has a majority of the 1st place votes, he will also have a plurality, and will win.

There are other voting methods that can violate this criteria – more on those later.

Bowl Election

Voters	49	48	3
1st choice	***Rose***	***H***	***F***
2nd choice	***Hula***	***S***	***H***
3rd choice	***Fiesta***	***O***	***S***
4th choice	***Orange***	***F***	***O***
5th choice	***Sugar***	***R***	***R***

So, the plurality method satisfies the majority criteria, but that's about all it has going for it.

Let's look at an example. The band is going to vote for which bowl game to perform at. Plurality says "Rose Bowl," but 51 folks put it in last place.

Hula has 48 first place and 52 second place, so it is probably a better choice.

If you compare hula to every other on a head-to-head basis, it is always the preferred choice (DO IT)

A candidate preferred over every other on a head-to-head basis is called a *Condorcet Candidate*. Not every election has one, but if there is one, it's a pretty good sign that the candidate represents the

choice of the voters better than any other.

Here, the Hula Bowl is a Condorcet Candidate.

In 1785, Condorcet introduced the principle now known as the *Condorcet Criterion*.

Condorcet Criterion

- If candidate *X* is preferred by the voters over each of the other candidates in a head-to-head comparison, then candidate *X* should be the winner of the election.

Remember this; we'll come back to it later.

Bowl Election

Voters	49	48	3 Dorseys
1st choice	R	H	F
2nd choice	H	S	H
3rd choice	F	O	S
4th choice	O	F	O
5th choice	S	R	R

Insincere (or strategic) voting. Here is the band election again. Notice that the three Dorsey brothers want to go to the Fiesta Bowl. But by the plurality method, the Rose Bowl wins.

Now the Dorseys are no dummies. They realize that there is no chance for the Fiesta Bowl to win, and they don't want to waste their votes. So if they switch their first and second place choices…

Bowl Election

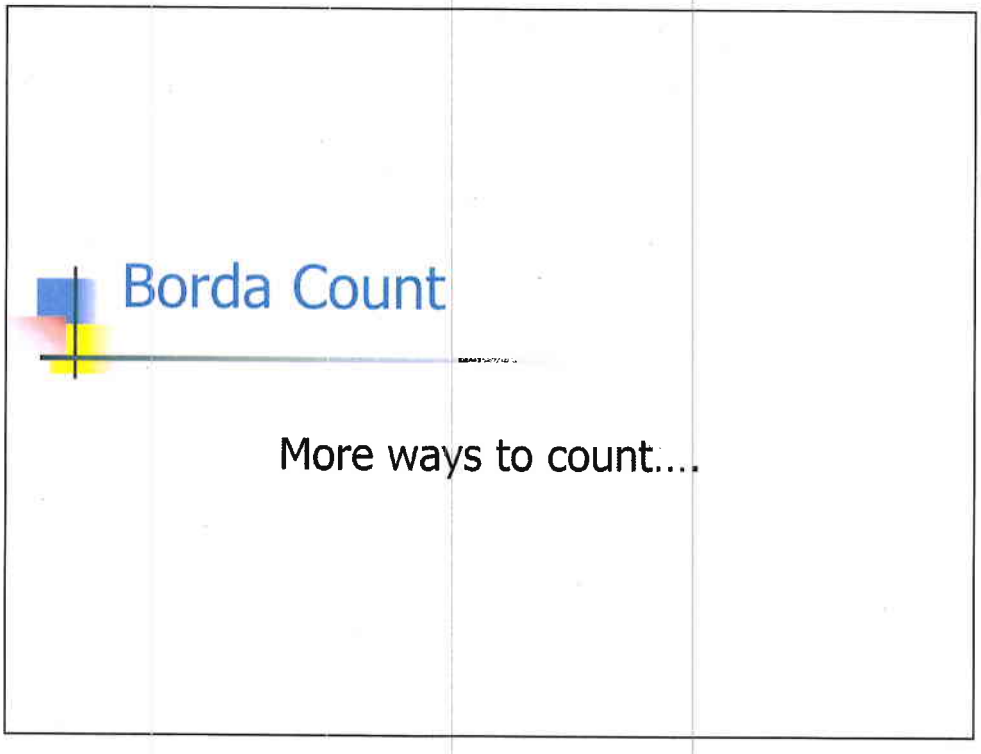
Voters	49	48	3 Dorseys
1st choice	R	H	H
2nd choice	H	S	F
3rd choice	F	O	S
4th choice	O	F	O
5th choice	S	R	R

…and put the Hula Bowl as their first choice, now the Hula Bowl wins.

All methods of voting can be manipulated, but the plurality method can be most easily manipulated. Some examples from American politics:

In the 2000 and 2004 presidential elections, Ralph Nader lost many votes from folks who realized that he couldn't be elected, and didn't want to waste their votes. This impacts the entire system. Because a party or independent candidate must receive 5% of the votes in order to qualify for federal funds for the next election, people who don't want to waste their votes can deny that funding next time.

Plurality method in a 2-party system often gives the voters very little real choice.



There are, in fact, other ways to count an election.

Math Club Election

Votes:	14	10	8	4	1
1st	***A***	***C***	***D***	***B***	***C***
2nd	***B***	***B***	***C***	***D***	***D***
3rd	***C***	***D***	***B***	***C***	***B***
4th	***D***	***A***	***A***	***A***	***A***

First of all, note that A is the winner under plurality.

The Borda Count method is a common way to determine the winner of an election. It is named after Jean-Charles de Borda.

Step 1. We prepare a preference schedule, and then start at the bottom and number the rows up…

Borda Count, Step 1

Votes:		14	10	8	4	1
4	1st	***A***	***C***	***D***	***B***	***C***
3	2nd	***B***	***B***	***C***	***D***	***D***
2	3rd	***C***	***D***	***B***	***C***	***B***
1	4th	***D***	***A***	***A***	***A***	***A***

Then we calculate points, like this…

Borda Count, Step 2

		Votes: 14	10	8	4	1
4	1st	A	C	D	B	C
3	2nd	B	B	C	D	D
2	3rd	C	D	B	C	B
1	4th	D	A	A	A	A

56

- Take the row number and multiply it times the number of votes and record the product

Borda Count, Step 2

	Votes:	14		10		8	4	1
4	1st	A	56	C	40	D	B	C
3	2nd	B		B		C	D	D
2	3rd	C		D		B	C	B
1	4th	D		A		A	A	A

We repeat this for every entry in the preference schedule

-
-
-
-

Borda Count, Step 2

	Votes:	14		10		8		4		1	
4	1st	**A**	56	**C**	40	**D**	32	**B**	16	**C**	4
3	2nd	**B**	42	**B**	30	**C**	24	**D**	12	**D**	3
2	3rd	**C**	28	**D**	20	**B**	16	**C**	8	**B**	2
1	4th	**D**	14	**A**	10	**A**	8	**A**	4	**A**	1

Let's stop for a moment to make certain you know where every number came from…

Borda Count, Step 2

Votes:		14		10		8		4		1	
4	1st	A	56	C	40	D	32	B	16	C	4
3	2nd	B	42	B	30	C	24	D	12	D	3
2	3rd	C	28	D	20	B	16	C	8	B	2
1	4th	D	14	A	10	A	8	A	4	A	1

A: 56 + 10 + 8 + 4 + 1 = 79 points

Now, go through and collect all the "A" points [runs automatically] and total them up. In this case, that's 79 points for "A."

Borda Count, Step 3

- A gets 56 + 10 + 8 + 4 +1 = 79
- B gets 42 + 30 + 16 + 16 + 2 = 106
- C gets 28 + 40 + 24 + 8 +4 = 104
- D gets 14 + 20 + 32 + 12 + 3 = 81

And we do the same thing for each of the candidates.

Borda Count

- So B is the winner!
  - Remember that A was the winner under the plurality method
- Seems to be an excellent way to count since all the voters' preferences are considered.

However...

- Lets consider another election:

Borda Count Problem

	Votes: 6			2		3	
4	1st	A	24	B	8	C	12
3	2nd	B	18	C	6	D	9
2	3rd	C	12	D	4	B	6
1	4th	D	6	A	2	A	3

→Who has a majority of 1st place votes? (A)

→Is there a Condorcet candidate? (A)

→Who is the Borda winner? (B)

- →B – 32
- →C – 30
- →A – 29
- →D – 19

Borda Count Problem

- Note 1: Candidate A has received a majority of First-Place votes.
- Note 2: Candidate A is preferred over each of the others head-to-head 6 to 5
- Note 3: Candidate B wins using the Borda count method, with 32 points against the next, candidate C, with 30.

Summarizing…

So this tells us….

Borda Count Problem

- So, the Borda Count method, in this case, violates both
  - The majority criterion, and
  - The Condorcet criterion.

…that the Borda count method *can* violate both the majority criterion and the Condorcet criterion.

⌘