Math 2568, Quiz #15, Fall 2013 Name KEY

Instructions: Show all work. Use exact answers unless specifically asked to round.

1. Use the Gram-Schmidt process to find an orthogonal basis for the space spanned by $\left\{\begin{bmatrix}1\\1\\0\\2\end{bmatrix}, \begin{bmatrix}3\\2\\-1\\0\end{bmatrix}, \begin{bmatrix}2\\-5\\1\\1\end{bmatrix}\right\}$.

$\vec{v_1} = \begin{bmatrix}1\\1\\0\\2\end{bmatrix} = \vec{w_1}$

$\vec{w_2} = \vec{v_2} - \frac{\vec{w_1}\cdot\vec{v_2}}{\|\vec{w_1}\|^2}\vec{w_1} = \begin{bmatrix}3\\2\\-1\\0\end{bmatrix} - \frac{3+2+0+0}{6}\begin{bmatrix}1\\1\\0\\2\end{bmatrix} = \begin{bmatrix}3-5/6\\2-5/6\\-1-0\\0-5/6*2\end{bmatrix} = \begin{bmatrix}13/6\\7/6\\-1\\-5/3\end{bmatrix}$

$\vec{w_2} = \begin{bmatrix}13\\7\\-6\\-10\end{bmatrix}$

$\vec{w_3} = \vec{v_3} - \frac{\vec{w_1}\cdot\vec{v_3}}{\|\vec{w_1}\|^2}\vec{w_1} - \frac{\vec{w_2}\cdot\vec{v_3}}{\|\vec{w_2}\|^2}\vec{w_2} = \begin{bmatrix}2\\-5\\1\\1\end{bmatrix} - \frac{2-5+0+2}{6}\begin{bmatrix}1\\1\\0\\2\end{bmatrix} - \frac{26-35-6+0}{354}\begin{bmatrix}13\\7\\-6\\-10\end{bmatrix}$

$= \begin{bmatrix}2-1/6+\frac{65}{118}\\-5-1/6+35/118\\1-0-15/59\\1-2/6-25/59\end{bmatrix} = \begin{bmatrix}422/177\\-862/177\\44/59\\43/177\end{bmatrix}$ $\vec{w_3} = \begin{bmatrix}422\\-862\\132\\43\end{bmatrix}$

$\left\{\begin{bmatrix}1\\1\\0\\2\end{bmatrix}, \begin{bmatrix}13\\7\\-6\\-10\end{bmatrix}, \begin{bmatrix}422\\-862\\132\\43\end{bmatrix}\right\}$

2. Use the information obtained in Problem #1 to find the QR factorization of $\begin{bmatrix}1&3&2\\1&2&-5\\0&-1&1\\2&0&1\end{bmatrix}$.

$A = QR$

$Q^TA = Q^TQR$

$Q^TA = R$

$Q = \begin{bmatrix}1/\sqrt{6} & 13/\sqrt{354} & 422/\sqrt{940,401}\\ 1/\sqrt{6} & 7/\sqrt{354} & -862/\sqrt{940,401}\\ 0 & -6/\sqrt{354} & 132/\sqrt{940,401}\\ 2/\sqrt{6} & -10/\sqrt{354} & 43/\sqrt{940,401}\end{bmatrix}$

$R \approx \begin{bmatrix}2.449 & 2.041 & -.408\\ 0 & 3.136 & -1.329\\ -.365 & -.608 & 5.495\end{bmatrix}$