Math 2568, Quiz #14, Fall 2013 Name KEY

Instructions: Show all work. Use exact answers unless specifically asked to round.

1. Determine if the two vectors are orthogonal, using the indicated inner product.

   a. $\vec{u} = \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}, \vec{v} = \begin{bmatrix} 4 \\ 2 \\ -2 \end{bmatrix}, \vec{u} \cdot \vec{v} = \vec{u}^T \vec{v}$

   $8 - 6 - 2 = 0$

   orthogonal

   b. $f(x) = \sin 3x, g(x) = \cos 3x, \langle f | g \rangle = \int_{-\pi}^{\pi} f(x)g(x)dx$

   $$\int_{-\pi}^{\pi} \sin 3x \cos 3x \, dx = \int_{-\pi}^{\pi} \frac{1}{2} \sin 6x \, dx = \frac{-1}{12} \cos 6x \Big|_{-\pi}^{\pi} =$$

   $$-\frac{1}{12}\left[\cos 6\pi - \cos(-6\pi)\right] = 0$$

   orthogonal

2. Given the orthogonal basis for the space R^3, $B = \left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ -2 \\ 5 \end{bmatrix} \right\}$, compute the following:

   a. A matrix U such that the columns are orthonormal.

   $$U = \begin{bmatrix} 1/\sqrt{6} & -2/\sqrt{5} & -1/\sqrt{30} \\ 2/\sqrt{6} & 1/\sqrt{5} & -2/\sqrt{30} \\ 1/\sqrt{6} & 0 & 5/\sqrt{30} \end{bmatrix}$$

   b. The vector $[\vec{x}]_B$ using the methods for an orthogonal basis. $\vec{x} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$

   $$c_1 = \frac{2+6+4}{6} = \frac{12}{6} = 2$$

   $$c_2 = \frac{-4+3+0}{5} = \frac{-1}{5}$$

   $$c_3 = \frac{-2-6+20}{30} = \frac{12}{30} = \frac{2}{5}$$

   $$[\vec{x}]_B = \begin{bmatrix} 2 \\ -1/5 \\ 2/5 \end{bmatrix}$$