Instructions: Show all work. Give exact answers.

1. For the discrete dynamical system given by $\overrightarrow{x_{k+1}} = A\overrightarrow{x_k}$ where $A = \begin{bmatrix} .5 & .6 \\ -.3 & 1.4 \end{bmatrix}$, find the following.

   a. The eigenvalues of the system.

   $$\lambda_1 = 1.1, \quad \lambda_2 = .8$$
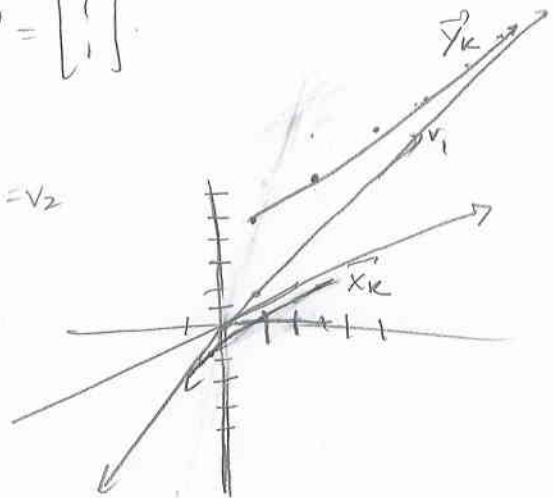
   b. Determine if the origin acts like a repeller, an attractor or a saddle point.

   Saddle point

   c. Find the eigenvectors of the system and plot them on a graph.

   $$\begin{bmatrix} -.6 & .6 \\ -.3 & .3 \end{bmatrix} \Rightarrow \begin{array}{l} -.3x_1 = -.3x_2 \\ x_1 = x_2 \\ x_2 = x_2 \end{array} \Rightarrow \overrightarrow{v_1} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

   $$\begin{bmatrix} -.3 & .6 \\ -.3 & .6 \end{bmatrix} \Rightarrow \begin{array}{l} -.3x_1 = -.6x_2 \\ x_1 = +2x_2 \\ x_1 = x_2 \end{array} \Rightarrow \begin{bmatrix} 2 \\ 1 \end{bmatrix} = v_2$$

   d. Find a 10-point trajectory from the points $\overrightarrow{x_0} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ and from $\overrightarrow{y_0} = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$. Sketch the trajectory on the graph with the eigenvectors. What do you notice?

   $\begin{bmatrix} 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 2.1 \\ 1.5 \end{bmatrix} \begin{bmatrix} 1.35 \\ .07 \end{bmatrix}, \begin{bmatrix} .717 \\ -.307 \end{bmatrix}, \begin{bmatrix} .1743 \\ -.6449 \end{bmatrix}, \begin{bmatrix} -.299 \\ -.955 \end{bmatrix}, \begin{bmatrix} -.722 \\ -1.247 \end{bmatrix}, \begin{bmatrix} -1.109 \\ -1.52 \end{bmatrix}, \begin{bmatrix} -1.47 \\ -1.8 \end{bmatrix}, \begin{bmatrix} -1.82 \\ -2.09 \end{bmatrix}, \begin{bmatrix} -2.16 \\ -2.37 \end{bmatrix}$

   $\begin{bmatrix} 1 \\ 5 \end{bmatrix}, \begin{bmatrix} 3.5 \\ 6.7 \end{bmatrix}, \begin{bmatrix} 5.77 \\ 8.33 \end{bmatrix}, \begin{bmatrix} 7.88 \\ 9.93 \end{bmatrix}, \begin{bmatrix} 9.9 \\ 11.5 \end{bmatrix}, \begin{bmatrix} 11.8 \\ 13.2 \end{bmatrix}, \begin{bmatrix} 13.8 \\ 14.89 \end{bmatrix}, \begin{bmatrix} 15.8 \\ 16.7 \end{bmatrix}, \begin{bmatrix} 17.95 \\ 18.6 \end{bmatrix}, \begin{bmatrix} 20.14 \\ 20.68 \end{bmatrix}, \begin{bmatrix} 22.48 \\ 22.91 \end{bmatrix}, \begin{bmatrix} 24.99 \\ 25.33 \end{bmatrix}$