Math 2568, Quiz #12, Fall 2013 Name KEY

Instructions: Show all work. Give exact answers.

1. Find the eigenvalues and eigenvectors of the following matrices. Which ones can be diagonalized?

a. $\begin{bmatrix} 1 & 2 \\ 6 & 2 \end{bmatrix}$

$\begin{bmatrix} 1-\lambda & 2 \\ 6 & 2-\lambda \end{bmatrix} \Rightarrow (1-\lambda)(2-\lambda)-12 \Rightarrow \lambda = 5, -2$ diagonalizable

$\begin{bmatrix} -4 & 2 \\ 6 & -3 \end{bmatrix}$ $6x_1 = 3x_2$, $x_1 = \frac{1}{2}x_2$, $x_2 = x_2 \Rightarrow \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \vec{v_1}$

$\begin{bmatrix} 3 & 2 \\ 6 & 4 \end{bmatrix} \Rightarrow 3x_1 = -2x_2$, $x_1 = -\frac{2}{3}x_2$, $x_2 = x_2 \Rightarrow \begin{bmatrix} -2 \\ 3 \end{bmatrix} = \vec{v_2}$

b. $\begin{bmatrix} 1 & 5 \\ -4 & 5 \end{bmatrix}$ $(1-\lambda)(5-\lambda)+20 \Rightarrow 3 \pm 4i = \lambda$

$\begin{bmatrix} 1-3-4i & 5 \\ -4 & 5-3-4i \end{bmatrix} = \begin{bmatrix} -2+4i & 5 \\ -4 & -2-4i \end{bmatrix}$

$-4x_1 = -(-2-4i)x_2$, $x_1 = \left(\frac{1}{2}-i\right)x_2$, $x_2 = x_2 \Rightarrow \begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \end{bmatrix} i = \vec{v_1}$

diagonalizable only if complex entries allowed

$\vec{v_2} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} i$

c. $\begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix}$ $(2-\lambda)(-\lambda)+1 \Rightarrow \lambda = 1$ repeated

$\begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix} \Rightarrow x_1 = -x_2$, $x_2 = x_2 \Rightarrow \vec{v_1} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ not diagonalizable

d. $\begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$ $(3-\lambda)(2-\lambda)-5 \Rightarrow \lambda^2 - 5\lambda + 1 = 0$ $\lambda = 5/2 \pm \frac{\sqrt{21}}{2}$

$\begin{bmatrix} 3-5/2-\frac{\sqrt{21}}{2} & -1 \\ -5 & 2-5/2-\frac{\sqrt{21}}{2} \end{bmatrix} = \begin{bmatrix} 1/2-\frac{\sqrt{21}}{2} & -1 \\ -5 & -1/2-\frac{\sqrt{21}}{2} \end{bmatrix}$

$-5x_1 = \left(\frac{1}{2}+\frac{\sqrt{21}}{2}\right)x_2$, $x_1 = \left(\frac{-1}{10}-\frac{\sqrt{21}}{10}\right)x_2$, $x_2 = x_2$ $\vec{v_1} = \begin{bmatrix} -1-\sqrt{21} \\ 10 \end{bmatrix}$

$\vec{v_2} = \begin{bmatrix} -1+\sqrt{21} \\ 10 \end{bmatrix}$

diagonalizable