Math 1149, Quiz #7, Fall 2013 Name KEY

Instructions: Show all work. Give exact answers unless specifically asked to round.

1. Solve the equation for all angles in $[0,2\pi)$: $2sin^2\theta = 3(1 - cos\theta)$.

$2(1-\cos^2\theta) = 3 - 3\cos\theta$

$2 - 2\cos^2\theta = 3 - 3\cos\theta$

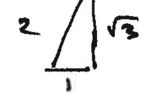
$0 = 2\cos^2\theta - 3\cos\theta + 1$

$(2\cos\theta - 1)(\cos\theta + 1) = 0$

$2\cos\theta - 1 = 0 \qquad \cos\theta = -1$

$\cos\theta = 1/2$

$\boxed{\theta = \pi/3,\ 5\pi/3,\ \pi}$



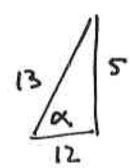
2. Verify the identity: $\frac{sec\theta}{1+sec\theta} = \frac{1-cos\theta}{sin^2\theta}$.

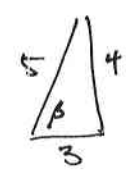
$$\frac{\frac{1}{\cos\theta}}{1+\frac{1}{\cos\theta}} \cdot \frac{\cos\theta}{\cos\theta} = \frac{1}{\cos\theta+1} \cdot \frac{\cos\theta - 1}{\cos\theta - 1} = \frac{\cos\theta - 1}{\cos^2\theta - 1} = \frac{\cos\theta - 1}{-1(1-\cos^2\theta)} =$$

$$\frac{-(\cos\theta - 1)}{\sin^2\theta} = \frac{1-\cos\theta}{\sin^2\theta} \ \checkmark$$

3. Use the sum and difference formulas to find the value of $tan\left(sin^{-1}\frac{5}{13} - cos^{-1}\frac{3}{5}\right)$.

$\alpha \qquad \beta$





$$\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta} = \frac{\frac{5}{12} - \frac{4}{3}}{1+\left(\frac{5}{12}\right)\left(\frac{4}{3}\right)} =$$

$$\frac{\frac{5}{12} - \frac{16}{12}}{1+\frac{5}{9}} = \frac{\frac{-11}{12}}{\frac{14}{9}} = \frac{-11}{12} \cdot \frac{9}{14} = \boxed{-\frac{33}{56}}$$

4. Use the half-angle formulas to find the exact value of $cos\left(-\frac{3\pi}{8}\right)$.

QIV

$\alpha = -3\pi/4 \Rightarrow$ QIII

$\cong 5\pi/4$

$\cos(5\pi/4) = -\frac{\sqrt{2}}{2}$

$$= +\sqrt{\frac{1+\frac{-1}{\sqrt{2}}}{2} \cdot \frac{\sqrt{2}}{\sqrt{2}}}$$

$$= \sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}} = \sqrt{\frac{2-\sqrt{2}}{4}} = \boxed{\frac{\sqrt{2-\sqrt{2}}}{2}}$$