Math 1149, Exam #4, Fall 2013 Name KEY

Instructions: Show all work. Round only when specifically asked to. In all other cases, use exact answers.

1. For the vectors $\vec{u} = 3\mathbf{i} - 4\mathbf{j}, \vec{v} = -\mathbf{i} + \mathbf{j}, \vec{w} = 2\mathbf{i} + 5\mathbf{j}$, find each of the following. (4 points each)

a. $\vec{v} + 2\vec{w}$

$$-\hat{i} + \hat{j} + 2(2\hat{i} + 5\hat{j}) = -\hat{i} + 4\hat{i} + \hat{j} + 10\hat{j} =$$
$$3\hat{i} + 11\hat{j}$$

b. $3\vec{u} - \vec{w}$

$$3(3\hat{i} - 4\hat{j}) - (2\hat{i} + 5\hat{j}) = 9\hat{i} - 2\hat{i} - 12\hat{j} - 5\hat{j} = 7\hat{i} - 17\hat{j}$$

c. $\|\vec{u} + \vec{w}\|$

$$\vec{u} + \vec{w} = 3\hat{i} - 4\hat{j} + 2\hat{i} + 5\hat{j} = 5\hat{i} + \hat{j}$$

$$\|\vec{u} + \vec{w}\| = \sqrt{5^2 + 1^2} = \sqrt{26}$$

d. $\vec{u} \cdot \vec{v}$

$$3 \cdot (-1) + (-4)(1) = -3 - 4 = -7$$

e. A unit vector in the direction of $\vec{v}$

$$\|\vec{v}\| = \sqrt{(-1)^2 + 1^2} = \sqrt{2}$$

$$\hat{v} = \frac{\vec{v}}{\|\vec{v}\|} = -\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}$$

f. Find the angle between $\vec{v}$ and $\vec{w}$.

$v \cdot w = (-1)(2) + (1)(5) = -2+5 = 3$

$\|\vec{v}\| = \sqrt{2}$ $\|\vec{w}\| = \sqrt{2^2+5^2} = \sqrt{4+25} = \sqrt{29}$

$\cos\theta = \frac{3}{\sqrt{2}\sqrt{29}} = \frac{3}{\sqrt{58}} \Rightarrow \theta = 1.1659$ radians or $66.8°$

g. Decompose the vector $\vec{w}$ into the projection onto $\vec{v}$ and it s orthogonal component. (6 points)

$proj_{\vec{v}} w = \frac{v \cdot w}{\|v\|^2}\vec{v} = \frac{3}{2}(-\hat{i}+\hat{j}) = -\frac{3}{2}\hat{i} + \frac{3}{2}\hat{j} = \vec{w_1}$

$\vec{w_2} = \vec{w} - \vec{w_1} = 2\hat{i} + 5\hat{j} - (-\frac{3}{2}\hat{i} + \frac{3}{2}\hat{j}) = (2+\frac{3}{2})\hat{i} + (5-\frac{3}{2})\hat{j} =$

$\frac{7}{2}\hat{i} + \frac{7}{2}\hat{j}$

(you can check by dot product that these are orthogonal)

2. Find a vector that points from the point (-1,4) to the point (6,2). Write it in the form $\vec{v} = a\mathbf{i} + b\mathbf{j}$. (4 points)

$6-(-1) = 7$

$2-4 = -2$

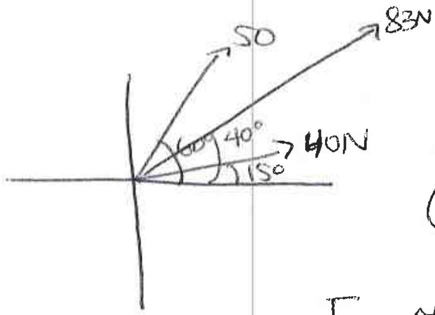
$\vec{v} = 7\hat{i} - 2\hat{j}$

3. Given a vector with magnitude $\|\vec{x}\| = 15$ and that it makes an angle of 315° with the positive x-axis, find the vector in the form $\vec{x} = a\mathbf{i} + b\mathbf{j}$. (5 points)

$15\cos 315°\,\hat{i} + 15\sin 315°\,\hat{j} =$

$\frac{15}{\sqrt{2}}\hat{i} - \frac{15}{\sqrt{2}}\hat{j}$.

4. Two forces, one of magnitude 40 N makes an angle with the x-axis of 15°, and a second with a magnitude of 50 N and makes an angle with the x-axis of 60°, pull on an object. Find the magnitude and direction of the resulting force. Draw a diagram of the forces. (8 points)



$40\cos 15^\circ \hat{i} + 40 \sin 15^\circ \hat{j} = \vec{F_1}$

$50\cos 60^\circ \hat{i} + 50 \sin 60^\circ \hat{j} = F_2$

$(40\cos 15^\circ + 50\cos 60^\circ)\hat{i} + (40\sin 15^\circ + 50\sin 60^\circ)\hat{j}$

$F_{TOTAL} \approx 63.637\hat{i} + 53.654\hat{j}$

$\|F_{TOTAL}\| = \sqrt{63.637^2 + 53.654^2} \approx 83.237$ N $\quad \theta = \tan^{-1}\left(\frac{53.654}{63.637}\right) = 40.1^\circ$ or $.7005$ rads.

5. Find the work done by a force of 3 pounds, acting in the direction 60° to the horizontal in moving an object from (0,0) to (1,7). (5 points)

$\vec{F} = 3\cos 60\hat{i} + 3\sin 60\hat{j}$

$\vec{d} = (1)\hat{i} + 7\hat{j}$

$\vec{F} \cdot \vec{d} = 3\cos 60 + 21\sin 60 \approx 19.6865$ ft-lbs.

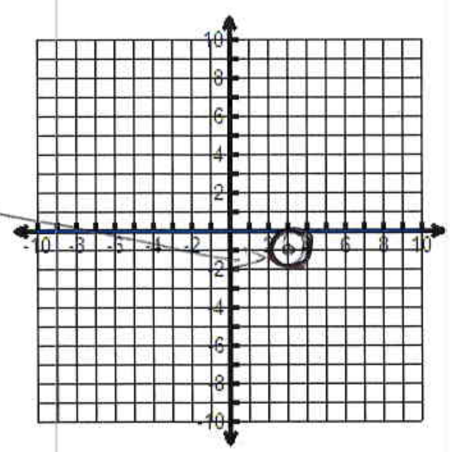
$\frac{3}{2} + \frac{21\sqrt{3}}{2} = \frac{3+21\sqrt{3}}{2} \approx$

6. Given the equation of the circle in general form given by $x^2 + y^2 - 6x + 2y + 9 = 0$, find the standard form of the circle and sketch the graph. Label the center on the graph. (8 points)

$(x^2 - 6x + 9) + (y^2 + 2y + 1) = \cancel{-9} \cancel{+9} + 1$

$(x-3)^2 + (y+1)^2 = 1$

$r = 1 \quad$ center $= (3,-1)$



7. Find an equation of the parabola with a vertex at (2,-3) and a focus at (2,-5). Sketch the graph. Label the vertex, the focus, the directrix and the axis of symmetry. (8 points)

$a=2$ opens down

$(x-2)^2 = -4(2)(y+3)$

$(x-2)^2 = -8(y+3)$

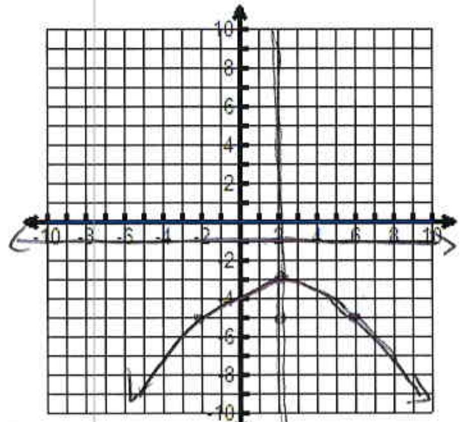
$y=-5$

$x-2 = \pm 4$

$x=6$

$x=-2$

directrix $Y=-1$



$X=2$ axis of symmetry

8. Find an equation of the ellipse with a center at (1,2), a focus at (4,2), and contains the point (1,3) on the minor axis. Sketch the graph. Label the center, the vertices, the foci, and the axis of symmetry. (8 points)

$C=3$ focus at $(-2,2)$

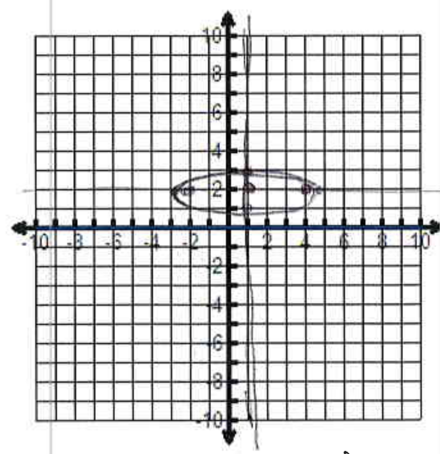
minor axis $(1,3),(1,1)$

$b=1$

$a=\sqrt{1^2+3^2}=\sqrt{10} \approx 3.2$

vertices $(1-\sqrt{10},2)$ $(1+\sqrt{10},2)$

$$\frac{(x-1)^2}{10}+\frac{(y-2)^2}{1}=1$$



$y=2$ major axis

$X=1$ minor axis

9. The equation of a hyperbola is given by $\frac{(x-2)^2}{4}-\frac{(y+3)^2}{9}=1$. Sketch the graph. Label the vertices, the foci, the center, the asymptotes and the transverse axis. (8 points)

center $(2,-3)$

$a=2$ $b=3$

transverse axis $\parallel$ to $x \Rightarrow y=-3$

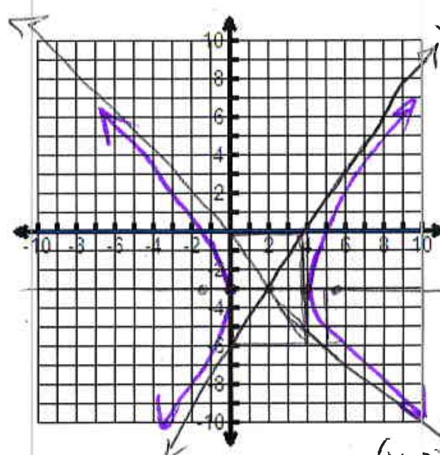
vertices $(0,-3),(4,-3)$

$\sqrt{a^2+b^2}=c \Rightarrow \sqrt{4+9}=\sqrt{13} \approx 3.6 = c$

foci $(2-\sqrt{13},-3)$ $(2+\sqrt{13},-3)$

asymptotes $(y+3)=\pm\frac{3}{2}(x-2)$

$y+3=\frac{3}{2}(x-2)$



transverse axis $y=-3$

$(y+3)=-\frac{3}{2}(x-2)$

10. Calculate the eccentricity for each of the conics in #7, 8, 9. (6 points)

a. #7: e= 1 (parabolas are always 1)

$e = \frac{c}{a}$

b. #8: e=

$\frac{3}{\sqrt{10}} \approx .9486$

c. #9: e=

$\frac{\sqrt{13}}{2} \approx 1.803$

11. Determine the type of graph in each polar conic equation below and find the eccentricity. (5 points each)

a. $r = \frac{3}{1 - sin\theta}$

$r = \frac{ep}{1 - e\sin\theta}$ $e = 1 \Rightarrow$ parabola

$p = 3$

b. $r = \frac{12}{4 + 8sin\theta}$

$\div 4/4$

$r = \frac{ep}{1 + 8\sin\theta} \Rightarrow \frac{3}{1 + 2\sin\theta} \Rightarrow e = 2$ hyperbola

$p = 3/2$

$e > 1$

c. $r = 2$ circle $e = 0$

d. $r = \frac{8}{4 + 3sin\theta}$ $\div 4/4$

$r = \frac{ep}{1 + e\sin\theta} \Rightarrow \frac{2}{1 + \frac{3}{4}\sin\theta}$

$0 < e < 1$

$e = 3/4$ ellipse

$p = 8/3$