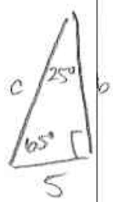
Math 1149, Exam #3, Fall 2013 Name KEY

Instructions: Show all work. Use exact values unless specifically asked to round in general computation problems. You may round to two decimal places in word problems unless directed to use a different level of accuracy. Be careful to use radian or degree mode as appropriate.

1. Given the following information about a right triangle, find the missing sides and acute angles A and B. You may round the sides to two decimal places, and the angles to one decimal place. (4 points each)

   a. $a = 5, A = 25°$



$\sin 25° = \frac{5}{c} \Rightarrow c = \frac{5}{\sin 25°} = 11.83$

$\tan 25° = \frac{5}{b} \Rightarrow b = \frac{5}{\tan 25°} = 10.72$

   b. $b = 4, c = 6$

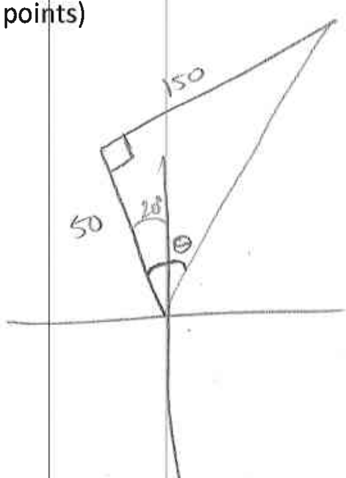
$36 - 16 = 2\sqrt{5}$

$\cos A = \frac{4}{6} \Rightarrow A = 48.2°$

$B = 41.8°$

$a = 2\sqrt{5} \approx 4.47$

2. An airplane leaves an airport bearing $N20°W$ flying at a speed of 50 mph for one hour. Then the pilot makes a 90° turn toward the northeast and continues flying at the same speed for another three hours. Find the bearing of the plane relative to the tower at the take-off airport. (8 points)



$\tan \theta = \frac{150}{50} \Rightarrow \theta = 71.6°$

$71.6 - 20 = 51.6°$

bearing $N\,51.6°\,E$

3. For each of the triangles below, use either the law of sines or the law of cosines as appropriate to solve for the missing sides of each triangle. Be wary of any possible cases where there could be two triangles and be sure to solve for both if they exist. If the triangle cannot be solved, indicate that there is no such triangle. (6 points each)

a. $a = 3, b = 7, A = 70°$

$$\frac{\sin 70}{3} = \frac{\sin B}{7} \Rightarrow \sin B = \frac{7 \sin 70}{3} = 2.19$$

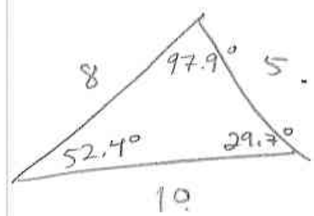
no triangle

b. $a = 8, b = 5, c = 10$

$$10^2 = 8^2 + 5^2 - 2 \cdot 8 \cdot 5 \cos C$$

$$\cos C = -.1375$$

$$C = 97.9°$$



$$\frac{\sin 97.9}{10} = \frac{\sin B}{5} \Rightarrow B = 29.7°$$

$$A = 180 - 97.9 - 29.7 = 52.4°$$

c. $a = 3, b = 7, A = 40°$

$$\frac{\sin 40}{3} = \frac{\sin B}{7}$$ no triangle

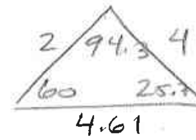
$$\sin B = 1.4998$$

d. $a = 4, b = 2, A = 60°$

$\frac{\sin 60}{4} = \frac{\sin B}{2} \Rightarrow B = 25.7°$ or $B = 154.3$ $+60$ > 180

$180 - 60 - 25.7 = 94.3° = C$

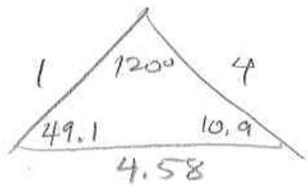
$\frac{\sin 60}{4} = \frac{\sin 94.3}{c} \Rightarrow c = \frac{4 \sin 94.3}{\sin 60} = 4.61$



e. $b = 4, a = 1, C = 120°$

$c^2 = 4^2 + 1^2 - 2 \cdot 1 \cdot 4 \cos 120$

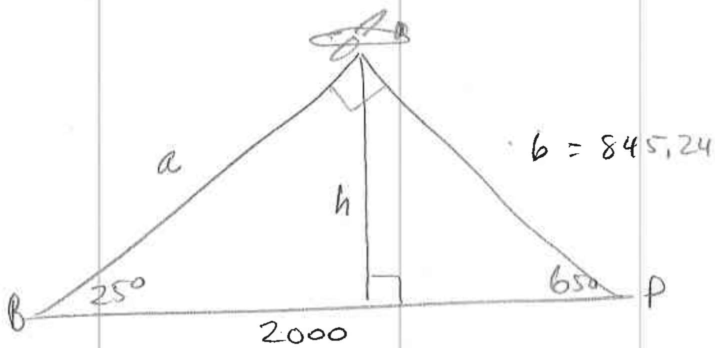
$c = \sqrt{21} = 4.58$



$\frac{\sin A}{4} = \frac{\sin 120}{4.58} \Rightarrow A = 49.1°$

$B = 180 - 120 - 49.1 = 10.9°$

4. Suppose that an airplane is flying overhead. Two people on the ground spot the plane and calculate the angle to the plane from their position at the same time. Ben sighted the plane at an angle of 25°, and Penny sighted the plane at an angle of 65°. They measure the distance between them and discover it is 2000 feet. How high is the plane flying at the time they noticed it? (8 points)



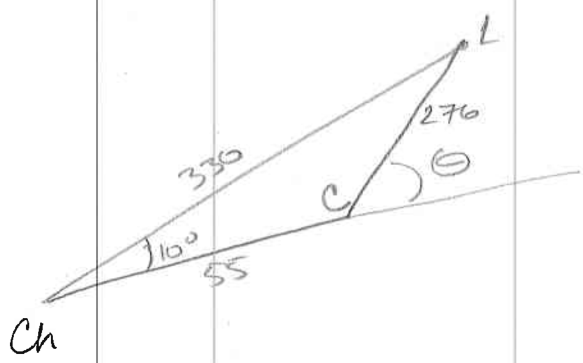
$b = 845.24$

$\frac{\sin 90}{2000} = \frac{\sin 25}{b} \Rightarrow b = 845.24$

$\sin 65 = \frac{h}{845.24}$

$h = 766.04$ feet

5. Chicago and Louisville are 330 miles apart as the crow flies. A plane leaves the Chicago airport flying at an average speed of 220 mph. The plane accidentally takes off at 10° off of the straight-line path, but it is discovered after 15 minutes in the air. The plane must make a turn to get back on a path for Louisville. Through what angle should the pilot turn the plane so that their path will intersect with Louisville? (8 points)



$b^2 = 330^2 + 55^2 - 2 \cdot 330 \cdot 55 \cos 10$

$b = 276.$

$\frac{\sin 10}{276} = \frac{\sin C}{330} \Rightarrow C = 12° \Rightarrow 168°$

$\boxed{\theta = 12°}$

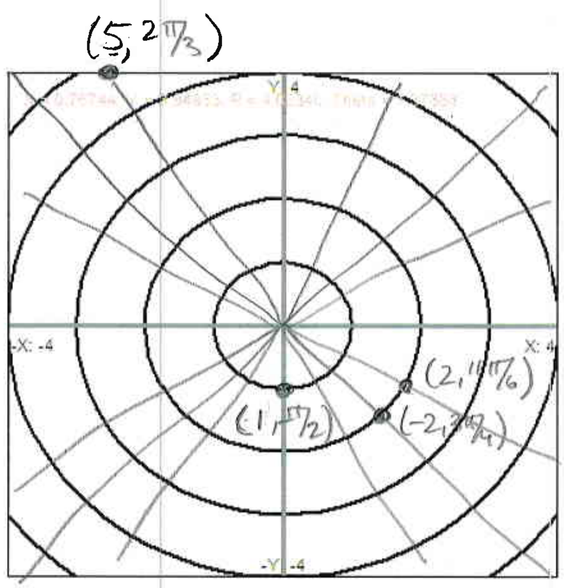
6. For each point in polar coordinates below, plot the point on the graph. Label each point. (1 point each)

a. $\left(5, \frac{2\pi}{3}\right)$

b. $\left(-2, \frac{3\pi}{4}\right)$

c. $\left(2, \frac{11\pi}{6}\right)$

d. $\left(1, -\frac{\pi}{2}\right)$



7. For each of the points in problem #6, convert the points into rectangular coordinates. (2 points each)
   a. $x = 5\cos 2\pi/3 = -5/2$ $\quad (-5/2, 5\sqrt{3}/2) \approx (-2.5, 4.33)$
   $y = 5\sin 2\pi/2 = 5\sqrt{3}/2$
   b. $x = -2\cos 3\pi/4 = -2(-\sqrt{2}/2) = \sqrt{2}$ $\quad (\sqrt{2}, -\sqrt{2}) \approx (1.41, -1.41)$
   $y = -2\sin 3\pi/4 = -\sqrt{2}$
   c. $x = 2\cos 11\pi/6 = 2\sqrt{3}/2 = \sqrt{3}$ $\quad (\sqrt{3}, -1) \approx (1.73, -1)$
   $y = 2\sin 11\pi/6 = 2(-1/2) = -1$
   d. $x = 1\cos(-\pi/2) = 0$ $\quad (0, -1)$
   $y = 1\sin(-\pi/2) = -1$

8. For the rectangular coordinate points below, convert them to polar form. You may use exact values for r or round to two decimal places. For θ, if the angle is a common one, use the exact expression, otherwise round to 4 decimal places in radians. (2 points each)
   a. $(-1,0)$ $\quad (1, \pi)$
   b. $(\sqrt{3},1)$ $\quad (2, \pi/6)$
   c. $(-8.3,-4.2)$ $\quad r = \sqrt{8.3^2+4.2^2} = 9.30$ $\quad (9.30, 3.61)$
   QIII $\tan^{-1}\left(\frac{-4.2}{-8.3}\right) + \pi = .4685 + \pi = 3.610$
   d. $(1.3,-2.1)$ $\quad r = \sqrt{1.3^2+2.1^2} = 2.47$ $\quad (2.47, -1.0165)$
   QIV $\tan^{-1}\left(\frac{-2.1}{1.3}\right) = -1.0165$

9. Convert the rectangular equations to polar equations, and convert the polar equations to rectangular equations. Solve for r (if possible) in the polar equations, and for y or a standard form in the rectangular ones. (3 points each)
   a. $x^2 = 4y$

$$\frac{r^2\cos^2\theta}{r} = \frac{4r\sin\theta}{r} \Rightarrow r = \frac{4\sin\theta}{\cos^2\theta} \Rightarrow r = 4\sec\theta\tan\theta$$

b. $y = -3$

$r\sin\theta = -3 \Rightarrow r = -3\csc\theta$

c. $r = sin\theta - cos\theta$ $r^2 = r\sin\theta - r\cos\theta$

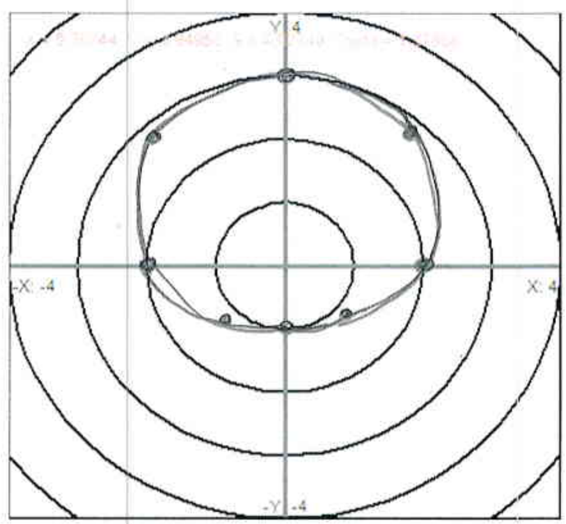
$x^2 + y^2 = y - x$

d. $r = 4$ $r^2 = 16$

$x^2 + y^2 = 16$

10. Graph the equation $r = 2 + sin\theta$ in polar coordinates on the graph below. You must plot and label at least 6 points on the graph. (9 points)

θ	$r = 2 + \sin\theta$
0	2
$\pi/4$	$2 + \sqrt{2}/2 \approx 2.71$
$\pi/2$	3
$3\pi/4$	$2 + \sqrt{2}/2 \approx 2.71$
π	2
$5\pi/4$	$2 - \sqrt{2}/2 \approx 1.29$
$3\pi/2$	1
$7\pi/4$	$2 - \sqrt{2}/2 \approx 1.29$



11. Write the complex number in polar form. (2 points each)

a. $4 - 4i$ $\quad r = 4\sqrt{2} \quad \theta = 7\pi/4 \quad 4\sqrt{2}[\cos 7\pi/4 + \sin 7\pi/4]$

QIV

b. $-2 + 3i$ $\quad r = \sqrt{13} \quad \theta = \tan^{-1}\left(\frac{3}{-2}\right) + \pi \simeq 2.1588$

QII

$\sqrt{13}[\cos 2.1588 + \sin 2.1588]$

12. Find $(1 - \sqrt{5}i)^8$ by converting to polar form and then back to standard form. (6 points)

$r = \sqrt{6}$

QIV

$\tan^{-1}(-\sqrt{5}) = -1.15 + 2\pi = 5.13$

$\left\{\sqrt{6}[\cos 5.1329 + \sin 5.1329]\right\}^8 = 6^4[\cos(41.06339) + \sin(41.06339)]$

$= -1264 - 286.22i$

13. Find all the complex fourth roots of $\sqrt{3} - i$ by converting to polar form, and then back to standard form. (6 points)

QIV

$r = 2 \quad \tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) = -\pi/6$

$\left\{2[\cos -\pi/6 + \sin \pi/6]\right\}^{1/4}$

$\sqrt[4]{2}[\cos 23\pi/24 + \sin 23\pi/24]$

$\sqrt[4]{2}[\cos(11\pi/24) + \sin(11\pi/24)]$

$-1.179 + .1552i$

$.1552 + 1.179i$

$\sqrt[4]{2}[\cos 35\pi/24 + \sin 35\pi/24]$

$\sqrt[4]{2}[\cos 47\pi/24 + \sin 47\pi/24]$

$-.1552 - 1.179i$

$1.179 - .1552i$