Math 1149, Exam #2, Fall 2013 Name KEY

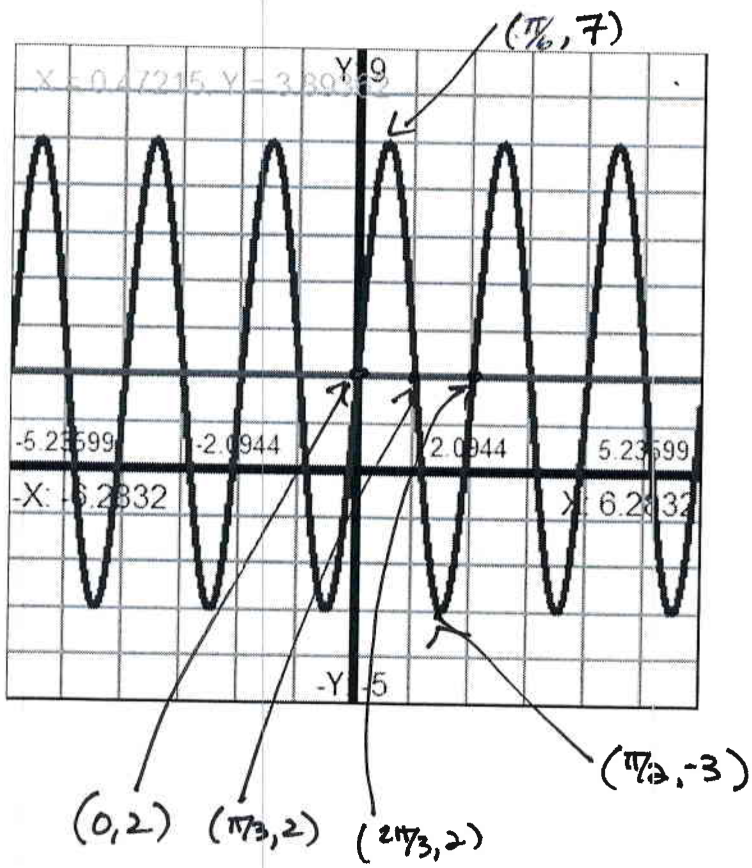
Instructions: Show all work. Use exact answers unless specifically asked to round. Answer all parts of each question. Be wary of problems in radian or degree mode: if no degree symbol is indicated, the angle is in radians.

1. For the function $f(x) = -2\cos\left(\pi x + \frac{3\pi}{2}\right) - 1$ state the following: (4 points)
   a. The amplitude 2
   b. The period $\frac{2\pi}{\pi} = 2$
   c. The phase shift $-\frac{3\pi}{2} \cdot \frac{1}{\pi} = -\frac{3}{2}$
   d. The midline -1

2. Graph the function $g(x) = 5\sin(3x) + 2$ below. Graph at least 2 periods. And label at least 5 points. (5 points)

$A = 5$
$T = \frac{2\pi}{3}$
midline $= 2$

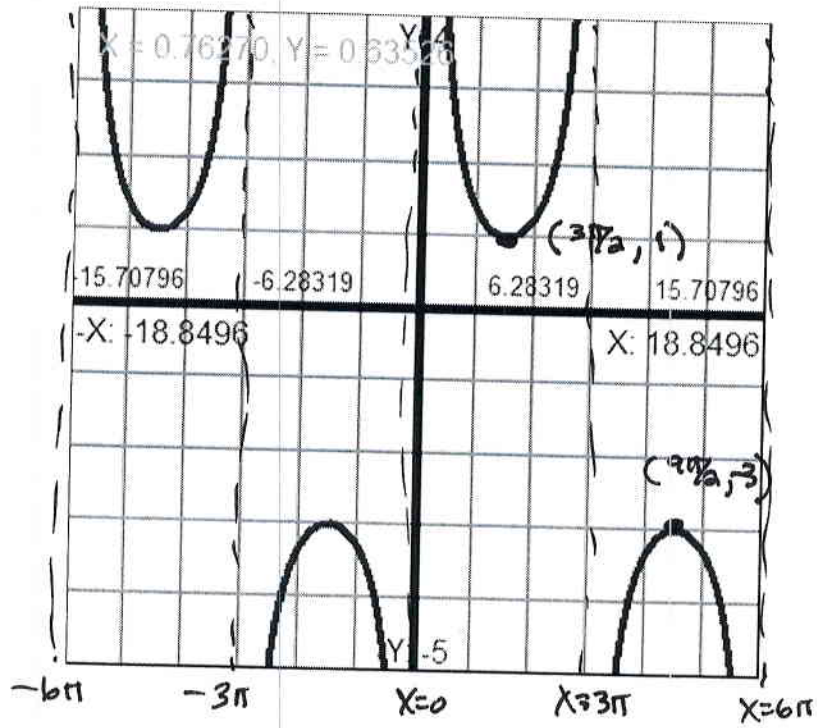
X	0	$\pi/6$	$\pi/3$	$\pi/2$	$2\pi/3$
X	0	$\pi/2$	π	$3\pi/2$	2π
Y	0	1	0	−1	0
Y	0	5	0	−5	0
Y	2	7	2	−3	2



3. Sketch the graph of the function $h(x) = 2\csc\left(\frac{1}{3}x\right) - 1$ below. Sketch at least 2 periods. Label at least one relative minima and one relative maxima, and any asymptotes. (5 points)

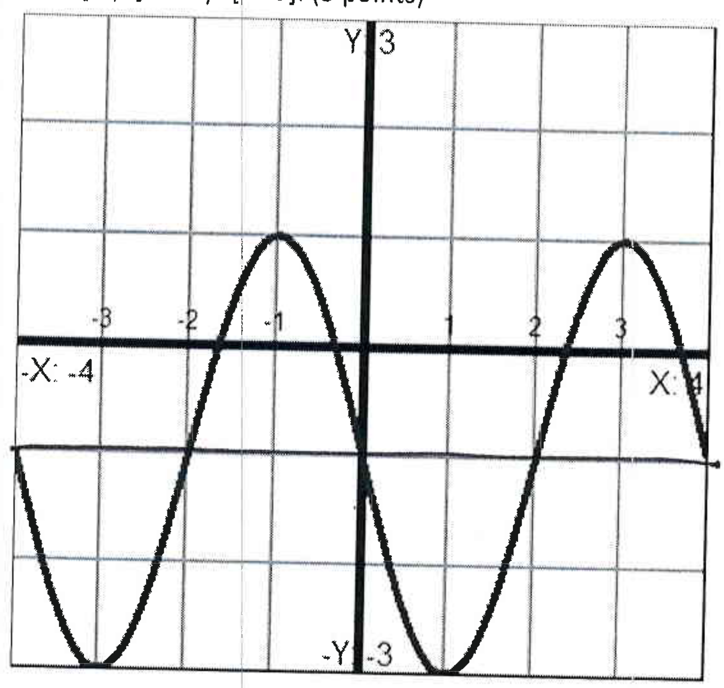
$T = 2\pi \cdot 3 = 6\pi$

x	0	$3\pi/2$	3π	$9\pi/2$	6π
x	0	$\pi/2$	π	$3\pi/2$	2π
y	un	1	un	-1	un
y	un	2	un	-2	un
y	un	1	un	-3	un



4. Find the equation of the trigonometric function graphed below. The axis lines appear every 1 unit on both axes, and the range on each axis is x:[-4,4] and y: [-3-3]. (3 points)

$y = -2\sin\left(\frac{\pi}{2}x\right) - 1$

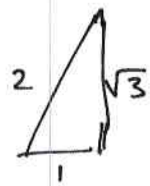


$\frac{2\pi}{4} = \frac{\pi}{2} = \omega$

5. Simplify each of the expressions below. Use exact answers. Give all angle values in radians. If the expression is undefined, state that. (2 points each)

a. $\arctan(-1)$ $-\pi/4$

b. $\arccos\left(-\frac{\sqrt{3}}{2}\right)$ $\frac{5\pi}{6}$



c. $sin^{-1}(0)$ 0

d. $\tan(tan^{-1}(\pi))$ π

e. $tan^{-1}\left(\tan\left(\frac{2\pi}{3}\right)\right)$ $-\pi/3$

f. $\sin(sin^{-1}(1.5))$ undefined

g. $cos^{-1}\left(\cos\left(\frac{2\pi}{3}\right)\right)$ $2\pi/3$

h. $csc^{-1}(2)$ $\pi/6$

i. $sec^{-1}(0)$ undefined

6. Use your calculator to approximate the values of the following expressions. Round your answers to three decimal places . (1 point each)

   a. $\arctan(5)$

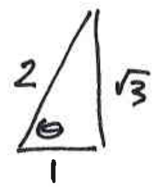
   1.373

   b. $\text{arcsec}(21)$ $= \cos^{-1}(1/21) \approx 1.523$

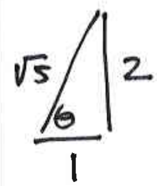
   c. $\cot^{-1}(-\sqrt{5})$ $= \tan^{-1}(-1/\sqrt{5}) = -.421$

7. Find the exact value of each expression. (3 points each)

   a. $\sec\left(\cos^{-1}\left(\frac{1}{2}\right)\right)$ $= 2$

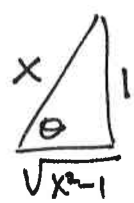


   b. $\tan\left(\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)\right)$ $= 2$

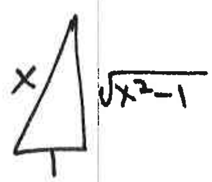


   c. $\cos^{-1}\left(\sin\left(\frac{7\pi}{6}\right)\right)$ $2\pi/3$

   d. $\cos(\csc^{-1}x)$ $= \dfrac{\sqrt{x^2-1}}{x}$



   e. $\tan(\sec^{-1}x)$ $= \sqrt{x^2-1}$



8. For the function $s(t) = 3\sin(2t+1)$, complete the following:

a. Find the domain and range of the function (2 points)

$D: (-\infty, \infty)$

$R: [-3, 3]$

restricted $\left(\left[\frac{-\pi/2}{2} - \frac{1}{2}, \frac{+\pi/2}{2} - \frac{1}{2}\right]\right)$

$\left[-\frac{\pi}{4} - \frac{1}{2}, +\frac{\pi}{4} - \frac{1}{2}\right]$

b. Find the inverse of the function. (2 points)

$y = 3\sin(2t+1) \Rightarrow t = 3\sin(2y+1) \Rightarrow t/3 = \sin(2y+1)$

$\arcsin(t/3) = 2y+1 \Rightarrow \arcsin(t/3) - 1 = 2y \Rightarrow$

$s^{-1}(t) = \frac{1}{2}\arcsin(t/3) - \frac{1}{2}$

c. State the domain and range of the inverse function. (2 points)

$D: [-3, 3]$

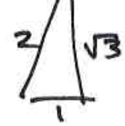
$R: \left[-\frac{\pi}{4} - \frac{1}{2}, \frac{\pi}{4} - \frac{1}{2}\right]$

9. Solve the following trigonometric equations for all values $0 \le \theta \le 2\pi$. Use exact answers. (3 points each)

a. $4\sin\theta + 3\sqrt{3} = \sqrt{3}$.

$\frac{4\sin\theta}{4} = \frac{-2\sqrt{3}}{4} \Rightarrow \sin\theta = -\frac{\sqrt{3}}{2}$

$\boxed{\theta = 4\pi/3, 5\pi/3}$



$\pi/3$ neg $\Rightarrow$ QIII, QIV

b. $\sin(3\theta) = -1$

$3\theta = 3\pi/2, 7\pi/2, 11\pi/2$

$\boxed{\theta = \pi/2, 7\pi/6, 11\pi/6}$

c. $1 + \sin\theta = 2cos^2\theta$

$1 + \sin\theta = 2(1 - \sin^2\theta) = 2 - 2\sin^2\theta$

$2\sin^2\theta + \sin\theta - 1 = 0 \qquad (2\sin\theta - 1)(\sin\theta + 1) = 0$

$2\sin\theta - 1 = 0 \Rightarrow \sin\theta = 1/2 \qquad \pi/3,\ 2\pi/3$

$\sin\theta + 1 = 0 \Rightarrow \sin\theta = -1 \qquad 3\pi/2$

$\boxed{\theta = \pi/3,\ 2\pi/3,\ 3\pi/2}$

d. $\sec\theta = \tan\theta + \cot\theta$ $)\cos\theta$

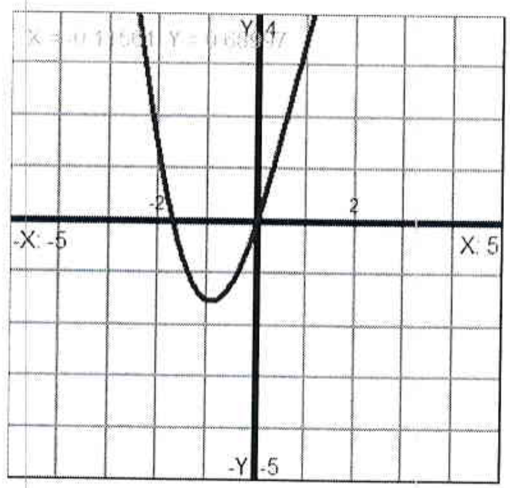
$\cancel{\frac{1}{\cos\theta}\cdot\cos\theta} = \frac{\sin\theta}{\cancel{\cos\theta}}\cdot\cancel{\cos\theta} + \frac{\cos\theta}{\sin\theta}\cdot\cos\theta$

$1 = \sin\theta + \frac{\cos^2\theta}{\sin\theta}$ $)\sin\theta \Rightarrow \sin\theta = \sin^2\theta + \cos^2\theta$

$\Rightarrow \sin\theta = 1 \qquad \boxed{\theta = \pi/2}$

10. Use your calculator to find all solutions of the equation $x^2 + 3\sin x = 0$. Round your answers to two decimal places. Sketch the graph. (3 points)

$X \approx -1.72,\ 0$



11. Show that each equation expresses an identity. (4 points each)

a. $\frac{\cos\theta}{1-\tan\theta} + \frac{\sin\theta}{1-\cot\theta} = \sin\theta + \cos\theta$

$\frac{\cos\theta}{1 - \frac{\sin\theta}{\cos\theta}}\cdot\frac{\cos\theta}{\cos\theta} + \frac{\sin\theta}{1 - \frac{\cos\theta}{\sin\theta}}\cdot\frac{\sin\theta}{\sin\theta} = \frac{\cos^2\theta}{\cos\theta - \sin\theta} + \frac{\sin^2\theta}{\sin\theta - \cos\theta}$

$= \frac{\cos^2\theta - \sin^2\theta}{\cos\theta - \sin\theta} = \frac{\cancel{(\cos\theta - \sin\theta)}(\cos\theta + \sin\theta)}{\cancel{\cos\theta - \sin\theta}} =$

$\cos\theta + \sin\theta$

b. $\frac{\sec\theta}{1-\sin\theta} = \frac{1+\sin\theta}{cos^3\theta}$

$$\frac{1}{\cos\theta} \cdot \frac{1}{1-\sin\theta} \cdot \frac{1+\sin\theta}{1+\sin\theta} = \frac{1+\sin\theta}{\cos\theta\,(1-\sin^2\theta)} =$$

$$\frac{1+\sin\theta}{\cos\theta\,(\cos^2\theta)} = \frac{1+\sin\theta}{\cos^3\theta}$$

c. $sec^4\theta - sec^2\theta = tan^4\theta + tan^2\theta$

$$\sec^2\theta\,(\sec^2\theta - 1) = (1+\tan^2\theta)\tan^2\theta =$$

$$\tan^2\theta + \tan^4\theta$$

d. $\frac{\cot\theta-\tan\theta}{\cot\theta+\tan\theta} = \cos(2\theta)$

$$\frac{\frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta}}{\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}} \cdot \frac{\sin\theta\cos\theta}{\sin\theta\cos\theta} = \frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin^2\theta}$$

$$= \cos^2\theta - \sin^2\theta = \cos 2\theta$$

12. Use the sum and difference formulas to find the exact value of each expression. (3 points each)

a. $\sin 20° \cos 10° + \cos 20° \sin 10°$

$\sin 30° = 1/2$

b. $\cos\left(\frac{5\pi}{12}\right) = \cos\left(\frac{3\pi}{12} + \frac{2\pi}{12}\right) = \cos\left(\frac{\pi}{4} + \frac{\pi}{6}\right) =$

$\cos\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{6}\right) - \sin\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{6}\right) =$

$\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2} = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$

13. Use the double angle or half-angle identities to find the values of each of the following expressions. (3 points)

a. $\sin\frac{\pi}{8}$ $\qquad \frac{\alpha}{2} = \frac{\pi}{8} \Rightarrow \alpha = \frac{\pi}{4}$

$= \sqrt{\frac{1-\cos\frac{\pi}{4}}{2}} = \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{\frac{1-\sqrt{2}}{2}}{2}} = \sqrt{\frac{1-\sqrt{2}}{4}} = \frac{\sqrt{1-\sqrt{2}}}{2}$

b. $\tan\frac{7\pi}{12}$ $\qquad \alpha/2 = 7\pi/12 \Rightarrow \alpha = 7\pi/6$

Q. →

$\frac{1-\cos(7\pi/6)}{\sin(7\pi/6)} = \frac{1-\left(-\frac{\sqrt{3}}{2}\right)}{-1/2} = \frac{1+\frac{\sqrt{3}}{2}}{-1/2} \cdot \frac{2}{2} = -(2+\sqrt{3})$

c. $\sin 15° \cos 15° = \frac{1}{2}\sin(2 \cdot 15°) = \frac{1}{2} \cdot \sin 30° = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

Bonus: Find the exact value of the expression: $\cos\left(\tan^{-1}\frac{4}{3} + \cos^{-1}\frac{12}{13}\right)$. (5 points)

$\tan^{-1}\frac{4}{3} = \alpha \qquad \cos^{-1}\frac{12}{13} = \beta$

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta =$

$\frac{3}{5} \cdot \frac{12}{13} - \frac{4}{5} \cdot \frac{5}{13} = \frac{36}{65} - \frac{20}{65} =$

$\boxed{\frac{16}{65}}$