Math 1116, Final Exam, Fall 2013 Name KEY

Instructions: Show all work. Use an appropriate number of decimal places. Many of the problems should use 4 decimal places throughout the calculation, though to you may asked to round to fewer places in the final answer. Probabilities involving small numbers should be exact and simplified. Probabilities involving large numbers can be rounded to 3 decimal places. Odds should always be expressed in terms of whole numbers.

1. An executive committee is to consist of 4 members: A president, vice president, secretary, and treasurer. If there are 8 men and 8 women available to serve on this committee, how many different committees can be formed? (6 points)

$16P4 = 43,680$

2. What is the probability that of the total number of possible committees, the one chosen ends up being all men? (assuming the selection is random) (8 points)

$\frac{8P4}{16P4} \approx .03846 = \frac{1}{26}$

3. What is the probability you will roll a 3 or a 4 when rolling a single die? (4 points)

$\frac{2}{6} = \frac{1}{3}$

4. What is the probability of obtaining all heads when flipping a coin 10 times? Is this event impossible? (5 points)

$\frac{1}{2^{10}} = \frac{1}{1024} \approx .0009765$

5. A country consisting of 4 states, A, B, C and D with populations given in the table below has 45 seats in the congress.

a. Apportion the seats using Jefferson's Method. You may use the modified quotas 170, 166, or 158. (20 points)

State	Population	Standard Quota	LQ	Modified Quota	LMQ	Final Apportionment
A	910	5.528	5	5.759	5	5
B	490	2.9769	2	3.101	3	3
C	280	1.701	1	1.77	1	1
D	5727	34.793	34	36.24	36	36
Total	7407		42		45	45

Standard Divisor: 164.6

b. What is the quota rule? (5 points)

The quota rule says states should be apportioned only their upper or lower quotas.

c. Has the quota rule been violated? Explain your answer. (5 points)

Yes. State D had an upper quota of 35 but received 36 seats under Jefferson's method.

6. An interplanetary senate has four planets in their system, and 75 seats to distribute in their legislature. Use Hamilton's Method to apportion the senate seats. (20 points)

State	Population	Standard Quota	LQ	+1	Final Apportionment
Illaria	44,912	25.02	25		25
Jasper	20,011	11.15	11		11
Kirk	38,236	21.30	21		21
Lubbock	31,474	17.53	17	+1	18
Total	134,633		74		75

Standard Divisor: 1795.1

7. For the data shown, apportion 28 seats to the 4 states using Huntington-Hill's Method. If you need a modified divisor try one of the following: 137, 146.7, 130. (20 points)

State	Population	Standard Quota	LQ	UQ	$\sqrt{LQ \cdot UQ}$	+1	Final Apportionment
Mississa	880	6.425	6	7	6.481		6
Nexis	1050	7.666	7	8	7.483	+1	8
Obala	730	5.330	5	6	5.477		5
Pimery	1175	8.579	8	9	8.485	+1	9
Total	3835						28

Standard Divisor: 136.96

no modified divisor needed

8. Consider the apportionment shown completed below. Determine which of the paradoxes is being illustrated. Explain why you chose the one you did. (10 points)

State	Population	Standard Quota	LQ	+1	Final Apportionment
Ulala	41	4.54	4	+1	5
Tempest	116	12.83	12	+1	13
Sondor	248	27.43	27		27
Ribbosa	47	5.2	5		5
Total	452		48		50

State	Population	Standard Quota	LQ	+1	Final Apportionment
Ulala	41	4.57	4		4
Tempest	116	12.93	12	+1	13
Sondor	248	27.65	27	+1	28
Ribbosa	47	5.24	5		5
Quenten	122	13.6	13	+1	14
Total	574		61		64

New States paradox
new state (Quenten) added; changed apportionment of other states

9. Find the probability of the following events.
   a. What is the probability of getting two heads in a row if you flip a coin twice? (5 points)

$\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

   b. What is the probability of getting a sum of 10 when rolling two dice? (5 points)

(4,6)
(6,4)
(5,5)

$\frac{3}{36} = \frac{1}{12}$

   c. What is the probability of having three girls in a family of 4 children? (5 points)

GGGB
GGBG
GBGG
BGGG

$\frac{4}{16} = \frac{1}{4}$

10. Define what it means for two events to be independent. (5 points)

the occurrence of one event does not effect the probability of the other

11. Give an example of two events that are independent, and two events that are dependent. (5 points)

independent: flip a coin, roll a die

dependent: pick a card from a deck, then another w/o replacement

12. Suppose a person rolls a standard die and pulls a card from a deck. (5 points each)
    a. What is the probability that the person will roll a 4 **OR** select a Jack?

$$\frac{1}{6} + \frac{4}{52} = \frac{19}{78} - \frac{1}{78} = \frac{18}{78} = \frac{3}{13}$$

    b. What is the probability that the person will roll a 4 **AND** select a Jack?

$$\frac{1}{6} \cdot \frac{4}{52} = \frac{1}{78}$$

13. If the probability of a certain event is 4/13. Find the probability that the event will **NOT** happen. (5 points)

$$1 - \frac{4}{13} = \frac{13}{13} - \frac{4}{13} = \frac{9}{13}$$

14. Find the value of the following expressions: (3 points each)

a. 7!

5040

b. 9C4 126

c. 11P5 55,440

d. $\binom{7}{2}$ 21

e. $\frac{5C3}{13C3}$ $= \frac{10}{280} = \frac{5}{143} \approx .0349$

15. How many ways can you do each of the following: (4 points each)

a. Pull a red marble followed by a yellow or a blue marble from a bowl containing 5 blue marbles, 5 green ones, 11white ones and 7 red ones?

$7 * (0+5) = 7*5 = 35$

(0 ↑ no yellows)

b. Get three of a kind from a standard card deck if your hand contains 5 cards?

$4C3 \cdot (13) * 48 * 47 = 29,328 \times 4 = 117,312$

(13 ↑ 3's of a kind; 48 * 47 ↖ can use remaining card of first type or 4 of a kind)

c. Form an 8 character password using lower case letters and numbers with only vowels and even numbers?

vowels = 5
even # = 5
10 options per character

$10^8 = 100,000,000$

16. What is the probability of choosing 3 face cards in a row from a standard card deck? (Face cards are the Jack, Queen and King.) (5 points)

$$\frac{12C3}{52C3} = \frac{220}{52C3} \approx .00995$$

17. If you walk into a room with 6 people in it, what is the probability that one of them will have been born on the same day of the week you were? (5 points)

$$1-\left(\frac{6}{7}\right)^6 = .6034$$

18. Suppose you are playing a dice game with a neighbor. He tells you that he will give you $1 if you roll a 1 or a 2, $4 if you roll a 6, and you pay him $2 if you roll another number. (10 points)

a. What is the expected value of the game?

$$\$1\left(\frac{2}{6}\right) + \$4\left(\frac{1}{6}\right) - \$2\left(\frac{3}{6}\right) = 0$$

b. Do you think the game is fair? Explain.

Yes. The expectation is to break even in the long run.

19. If the odds **FOR** an event are 3:2, what are the odds **AGAINST** the event? (3 points)

2:3

20. What are the odds of having 2 girls in a family of three children? (5 points)

3:5

21. If the odds against an event are 7:3, what is the probability of the event? (5 points)

$\frac{3}{10}$

22. For the data in the table below, compute the requested probabilities.

	Owns a Power Washer	Doesn't Own a Power Washer	Total
Female	11	73	84
Male	52	91	143
Total	63	164	227

a. What is the probability that someone who owns a power washer is female? (4 points)

$\frac{11}{63}$

b. What is the probability that a man owns a power washer? (4 points)

$\frac{52}{143}$

c. What is the probability that you would randomly select a person who is a woman without a power washer (from the entire group)? (4 points)

$\frac{73}{227}$

d. Is the probability of owning a power washer the same for women as for men? Are these events independent? (5 points)

No. These events are not independent.