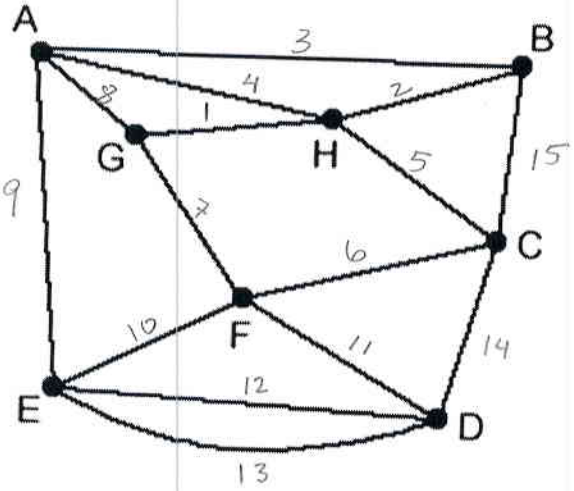
Math 1116, Exam #2, Fall 2013 Name KEY

Instructions: Show all work. Indicate routes around circuits (paths) as required by each problem.

1. An executive committee is to consist of 4 members: A president, vice president, secretary, and treasurer. If there are 8 men and 8 women available to serve on this committee, how many different committees can be formed? (8 points)

$16P4 = 43,680$

2. For the graph given below, determine if there is an Euler circuit, Euler path, or neither and explain your answer. If there is an Euler circuit, find one that starts at A. If there is an Euler path, find one. If there is neither an Euler circuit nor Euler path, then give an optimal eulerization of the graph. Number the edges as you use them. (20 points)



There is an Euler path; 2 vertices (G & B) are odd; the rest are even.

Starting at G → ending at B

G H B A H C F, G, A E, F, D E D C B

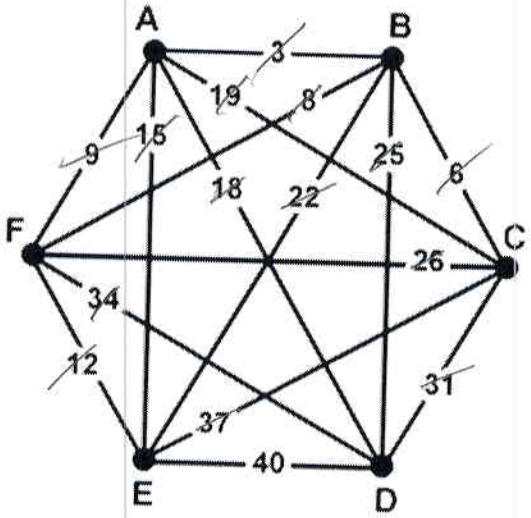
Answers will vary

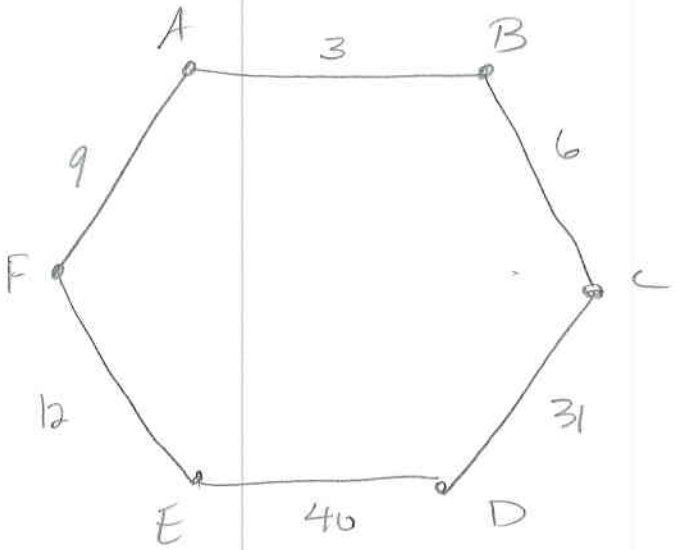
3. Use the nearest-neighbor algorithm starting at A to find a Hamilton circuit for the weighted graph shown below. State the total weight of the circuit that you find. (20 points)

A - B - C - F - E - D - A

3 + 6 + 26 + 12 + 40 + 18 =

105



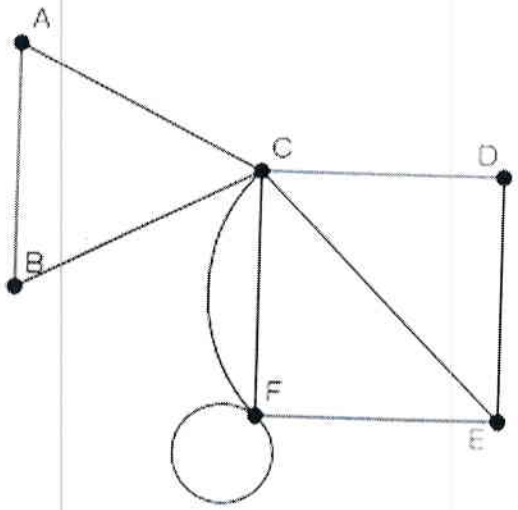


4. Use the Cheapest Link (Sorted Edges) Algorithm on the same graph. State the weight of the final circuit. Which is the better option in this instance? (15 points)

3 + 6 + 9 + 31 + 12 + 40 = 101

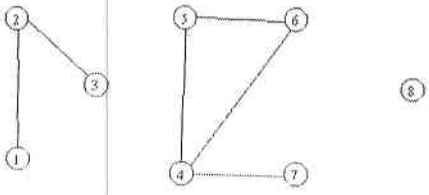
Cheapest Link did a slightly better job in this instance

5. For the graph shown, list the degree of each vertex. (6 points)



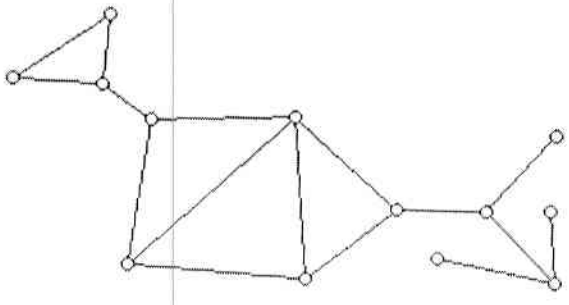
vertex	degree
A	2
B	2
C	6
D	2
E	3
F	5

6. State whether the graphs below are connected or not connected. (5 points each)



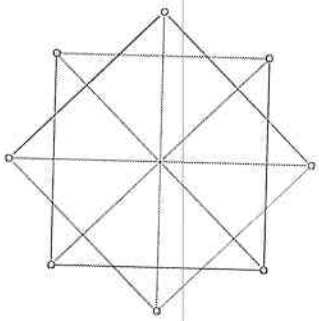
not connected

a.



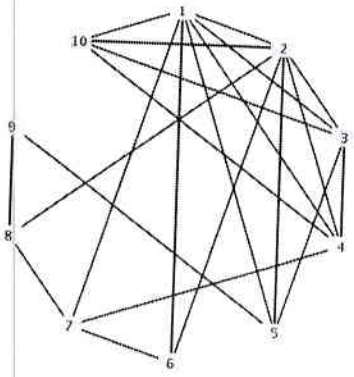
Connected

b.



not connected

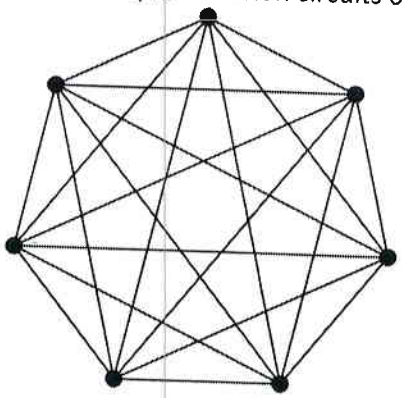
c.



Connected

d.

7. Determine which of the following graphs are complete. For the one(s) that is(are), find the number of unique Hamilton circuits of the graph. (8 points)



a.

complete

7 vertices

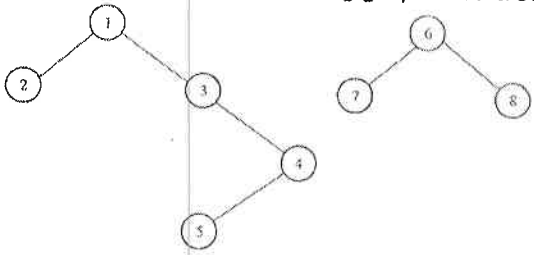
$\frac{(7-1)!}{2} = \frac{6!}{2} = \frac{720}{2} =$

360

b.

not complete

8. Determine which of the following graphs are trees. (6 points)

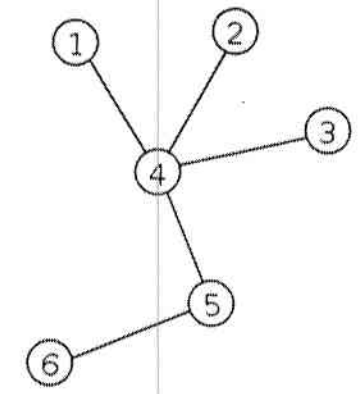


a.

not connected

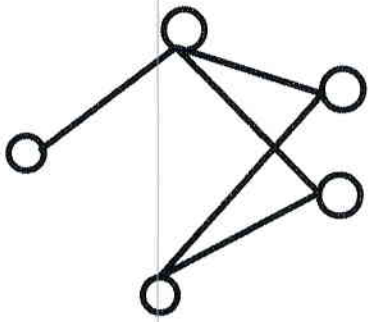
∴ not a tree

b.



tree

c.



at least on circuit

not a tree

9. A tree with 11 vertices should have exactly how many edges? (5 points)

10 edges.

10. In your own words, explain what a bridge is. How does it related to trees and Euler circuits? Give examples to illustrate. (10 points)

a bridge is an edge connecting a graph in such a way that if the edge is removed, it makes the graph disconnected.

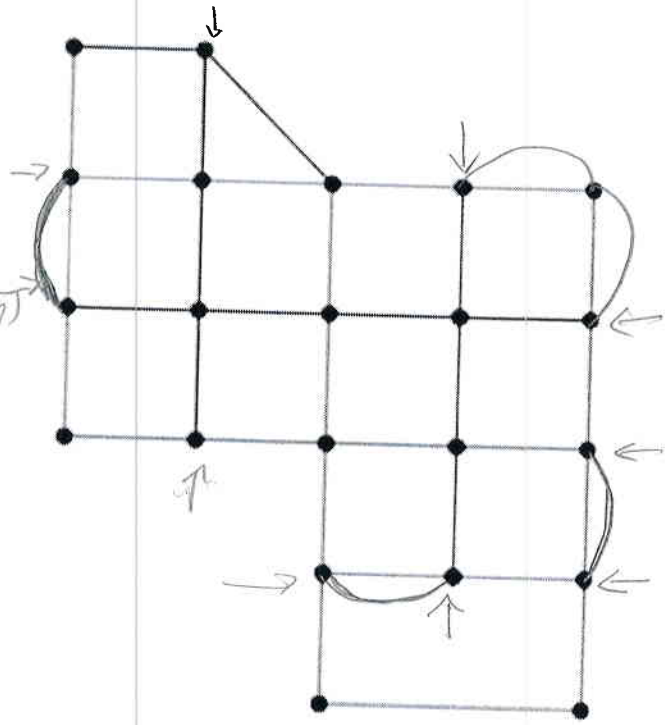
all edges in a tree are bridges.

bridges are used in Fleury's algorithm to find an Euler circuit.

11. Semi-eulerize the graph below. Is this the optimal semi-eulerization? What is the theoretically minimum number of edges you would need to add to the original graph to semi-eulerize it? (15 points)

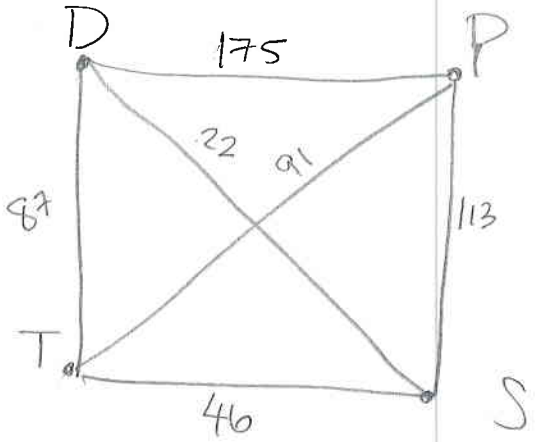
10 odd vertices
- 2
8/2 = 4 edges
min (theoretically)

5 edges minimum
for optimum

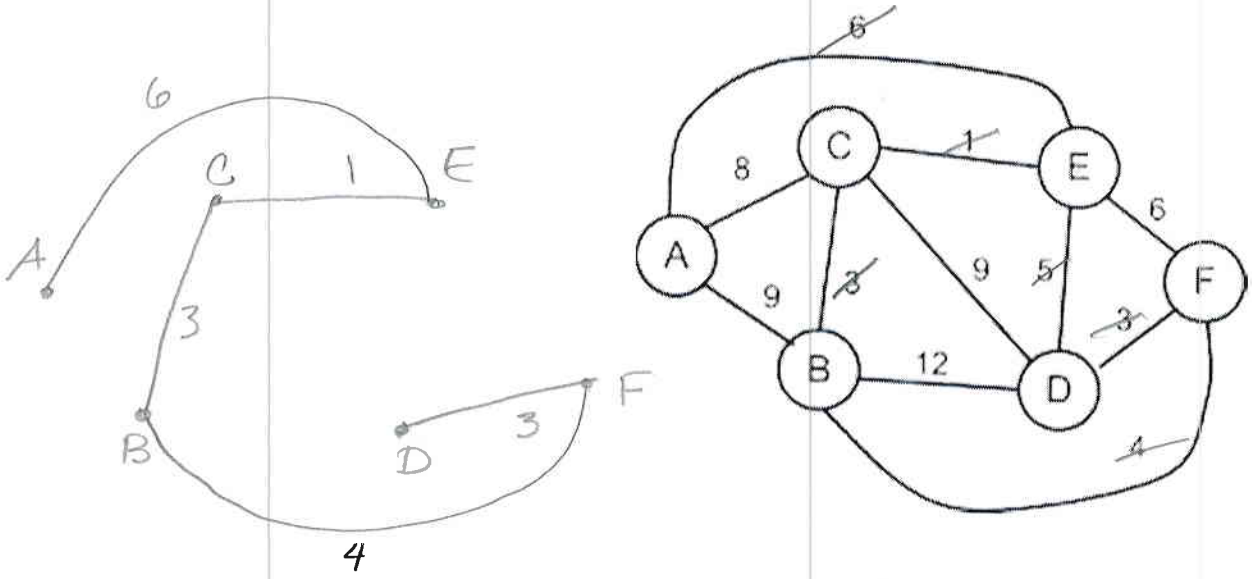


12. For the table below, draw the graph that would represent the information in the table. Be sure to include weights on the appropriate edges. (10 points)

	Dooby	Parma	Sidney	Topher
Dooby	-	175	22	87
Parma	175	-	113	91
Sidney	22	113	-	46
Topher	87	91	46	-



13. Use Kruskal's algorithm to find the minimal spanning tree of the graph below. (20 points)



$6+1+3+4+3=17$

14. State whether each of the following algorithms is efficient? (4 points each)
    a. Brute Force NO
    b. Nearest Neighbor YES
    c. Cheapest Link (Sorted Edges) YES
    d. Kruskal's YES

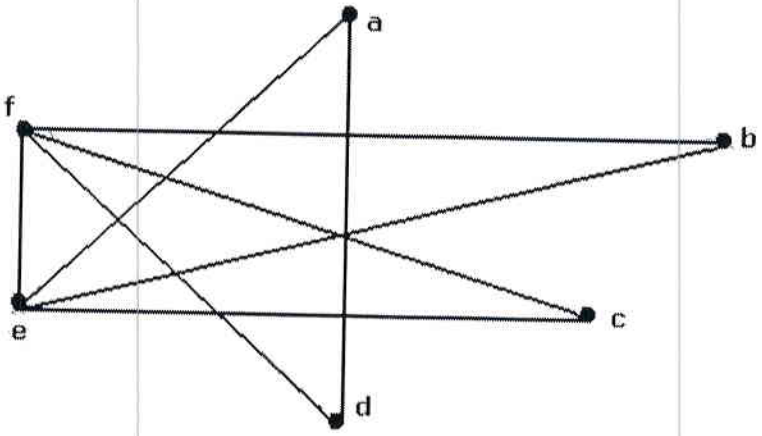
15. State whether each of the following algorithms is optimal? (4 points each)
    e. Brute Force YES
    f. Nearest Neighbor NO
    g. Cheapest Link (Sorted Edges) NO
    h. Kruskal's YES

16. If a connected graph has 10 vertices and 13 edges, how many need to be removed to have a spanning tree? (5 points)

tree needs $10-1=9$

$13-9=4$ need to be removed

17. Determine if the graph below has a Hamilton circuit. If it does, label the edges you would use to find it. (10 points)



No Hamilton circuit.