Math 1116, Exam #1, Fall 2013 Name KEY

Instructions: Show all work. Use correct notation where applicable. Answers have some justification in most cases.

For problems 1-3 write the answers in permutation or combination notation, if appropriate, or as an exponent. Evaluate each expression and give an exact answer unless your answer exceeds a million; if it is, then give the answer in scientific notation with three significant digits.

1. What are the number of ways you can choose marbles from a bowl in the order blue, green, white, red, green (in that order) if the bowl has 11 red marbles, 8 green marbles, 12 blue marbles and 3 white marbles? (8 points)

$\underset{\text{blue}}{12} \cdot \underset{\text{green}}{8} \cdot \underset{\text{white}}{3} \cdot \underset{\text{red}}{11} \cdot \underset{\text{green}}{7} = 22,176$

2. How many ways can the Glee Club choose a director, a prop manager and a lighting manager for their new play if there are 19 members of the club? (8 points)

$19P3 = 5814$

3. How many ways can the Glee Club choose a chorus line of three members if there are 11 women and 8 men on the club and the chorus line must be all men or all women? (8 points)

$11C3 + 8C3 = 165 + 56 = 221$

female choruses; male choruses

4. Calculate the following expressions. (6 points each)

a. $5P3$ 60

b. $11C8$ 165

c. $\binom{10}{4} = 210$

For problems 5-9, refer to the preference ballot show below.

# of Votes	20	18	15	11	6
1st choice	B	K	G	T	T
2nd choice	G	G	T	G	K
3rd choice	K	B	K	K	G
4th choice	T	T	B	B	B

5. Find the winner of the above election using the plurality method. Then rank the remaining candidates using that method. (15 points)

B=20
K=18
G=15
T=17

B - 1st place - winner!
K - 2nd place
T - 3rd place
G - 4th place

6. Calculate the winner of the election using the Borda Count method. (20 points)

B = 20(4) + 18(2) + 15(1) + 11(1) + 6(1) = 148
G = 20(3) + 18(3) + 15(4) + 11(3) + 6(2) = 219
K = 20(2) + 18(4) + 15(2) + 11(2) + 6(3) = 182
T = 20(1) + 18(1) + 15(3) + 11(4) + 6(4) = 151

G is the winner!

7. Calculate the winner of this election using the Plurality with Elimination method. (15 points)

Round 1
B=20
K=18
~~G=15~~
T=17
no majority
needs 36

Round 2
B=20
~~K=18~~
T=15+17=32
no majority

Round 3
B=20+18=38
T=32

B wins

8. Calculate the winner of the election using the method of pairwise comparisons. (15 points)

B-G	B: 20	G: 18+15+1+6 ✓	G
B-K	B: 20	K: 18+15+11+6 ✓	K
B-T	B: 20+18 ✓	T: 15+11+6	B
G-K	G: 20+15+11 ✓	K: 18+6	G
G-T	G: 20+18+15 ✓	T: 11+6	G
K-T	K: 20+18 ✓	T: 15+11+6	K

B - 1
G - 111
K - 11
T -

G wins

9. Does the Borda count method violate the Condorcet criterion in this election? (5 points)

No since Borda count and pairwise comparison yield the same result

10. Define the Independence of Irrelevant Alternatives criterion. (7 points)

this criterion says that if a losing candidate (such a T) drops out of the election, it should not effect the winner of the election

in our election if T drops out, the winner of the Plurality w/ Elimination method changes (to G)

11. Explain in your own words what Arrow's Impossibility Theorem says. (8 points)

it says that it is impossible to satisfy all voting criteria with any single method.

12. Given the weighted voting system [60: 21, 15, 14, 10, 8, 5, 5, 2], answer the following questions.
    a. What is the quota in this system. (5 points)

60

    b. What is the smallest and largest possible quota for this system? (5 points)

Smallest = 41

largest = 80

    c. How many players are in this system? (5 points)

8

    d. Does the system have any dictators? Players with veto power? Dummies? If so, who are they? (9 points)

no dictators

P1 has veto power

no dummies

13. Give an example of a weighted voting system with a dictator. (5 points)

answers will vary

[10: 10, 2, 2, 1]

14. Consider the weighted voting system [8: 4,3,2,2], calculate the Banzhaf power distribution for this system. (20 points)

	critical	power
{P1, P2, P3, P4}		
{P1, P2, P3}	P1: 4	40%
{P1, P2, P4}	P2: 2	20%
{P1, P3, P4}	P3: 2	20%
	P4: 2	20%
	10	

15. For the weighted voting system [8: 4,3,2,2], calculate the Shapley-Shubik power distribution. (20 points)

			pivotal	power
<P1, P2, P3, P4>	<P3, P1, P2, P4>	P1:	12	50%
<P1, P2, P4, P3>	<P3, P1, P4, P2>	P2:	4	17%
<P1, P3, P2, P4>	<P3, P2, P1, P4>	P3:	4	17%
<P1, P3, P4, P2>	<P3, P2, P4, P1>	P4:	4	17%
<P1, P4, P2, P3>	<P3, P4, P1, P2>		24	
<P1, P4, P3, P2>	<P3, P4, P2, P1>			
<P2, P1, P3, P4>	<P4, P1, P2, P3>			
<P2, P1, P4, P3>	<P4, P1, P3, P2>			
<P2, P3, P1, P4>	<P4, P2, P1, P3>			
<P2, P3, P4, P1>	<P4, P2, P3, P1>			
<P2, P4, P1, P3>	<P4, P3, P2, P1>			
<P2, P4, P3, P1>	<P4, P3, P1, P2>			

16. Consider a weighted voting system with 8 players. How many sequential coalitions are there? (5 points)

$8! = 40,320$

17. Consider a standard deck of 52 cards. How many ways are there to choose the following cards in sequence without replacement: a heart, a spade, a black card, a diamond, a heart. (8 points)

$\frac{13}{\heartsuit} \cdot \frac{13}{\spadesuit} \cdot \frac{25}{\text{Bl.}} \cdot \frac{13}{\diamondsuit} \cdot \frac{12}{\heartsuit} = 659,100$

18. If a license plate in a certain state can have 3 letters, followed by 3 numbers, followed by 3 letters, and the letter O may not be used, how many license plates of this form are possible? (8 points)

$\underbrace{25 \cdot 25 \cdot 25}_{\text{letter} \neq O} \cdot \underbrace{10 \cdot 10 \cdot 10}_{\#'s} \cdot \underbrace{25 \cdot 25 \cdot 25}_{\text{letters} \neq O} = 25^6 \cdot 10^3 =$

2.44×10^{11}