Instructions: Show all work. Use exact answers unless specifically asked to round. Be sure to complete all parts of each problem.

1. Find $\frac{\partial z}{\partial y}$ for the function $x^2 - 2y - z^2 + x^2yz^2 = 20$ implicitly. (You should do it the "long" way, but you may check your work by the short-cut formula.)

$$0 - 2 - 2zz_y + x^2\left(z^2 \cdot 1 + y \cdot 2z\, z_y\right) = 0$$

$$-2 - 2zz_y + x^2z^2 + 2x^2yz \cdot z_y = 0$$

$$-2zz_y + 2x^2yz \cdot z_y = 2 - x^2z^2$$

$$z_y\left(2x^2yz - 2z\right) = 2 - x^2z^2$$

$$\boxed{z_y = \frac{2 - x^2z^2}{2x^2yz - 2z}}$$

Check $F(x,y,z) = x^2 - 2y - z^2 + x^2yz^2 - 20$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{-2 + x^2z^2}{-2z + 2x^2yz} \quad \text{checks} \checkmark$$

2. Suppose that a production function is given by $P = \frac{kl}{3k+5l}$. Find the marginal productivity functions (i.e. $\frac{\partial P}{\partial k}, \frac{\partial P}{\partial l}$).

$$\frac{\partial P}{\partial k} = \frac{l(3k+5l) - 3(kl)}{(3k+5l)^2} = \frac{3kl + 5l^2 - 3kl}{(3k+5l)^2} = \boxed{\frac{5l^2}{(3k+5l)^2}}$$

$$\frac{\partial P}{\partial l} = \frac{k(3k+5l) - 5(kl)}{(3k+5l)^2} = \frac{3k^2 + 5kl - 5kl}{(3k+5l)^2} = \boxed{\frac{3k^2}{(3k+5l)^2}}$$