Math B1131, Quiz #7, Fall 2012 Name KEY

Instructions: Show all work. Use exact answers unless specifically asked to round.

1. Integrate.
   a. $\int \frac{x-1}{(x+2)(x-3)}dx$ by partial fractions.

$\frac{A}{x+2} + \frac{B}{x-3} = \frac{A(x-3)+B(x+2)}{(x-3)(x+2)}$

$Ax + Bx - 3A + 2B = x - 1$

$Ax + Bx = x \qquad -3A + 2B = -1$

$A + B = 1$

$A = -B + 1$

$-3(-B+1) + 2B = -1$

$3B - 3 + 2B = -1$

$5B = 2 \qquad B = 2/5 \qquad A = -2/5 + 1 = 3/5$

$\int \frac{3/5}{x+2} + \frac{2/5}{x-3}\,dx$

$\boxed{\frac{3}{5}\ln|x+2| + \frac{2}{5}\ln|x-3| + C}$

   b. $\int \frac{3x^2}{\sqrt{5x^2-11}}dx$ by using the integral tables in your textbook. State the formula number you use and any substitutions needed. Then apply the formula.

$\#29 \int \frac{u^2}{\sqrt{u^2 \pm a^2}} = \frac{1}{2}\left(u\sqrt{u^2 \pm a^2} \mp a^2 \ln\left|u + \sqrt{a^2 \pm a^2}\right|\right) + C$

$a = \sqrt{11}$

$u = \sqrt{5}x$

$du = \sqrt{5}dx$

$3\int \frac{x^2}{\sqrt{(\sqrt{5}x)^2 - (\sqrt{11})^2}} \cdot \frac{5\sqrt{5}}{\sqrt{5}\cdot 5} dx = \frac{3}{5\sqrt{5}} \int \frac{(\sqrt{5}x)^2 (\sqrt{5})}{\sqrt{(\sqrt{5}x)^2 - \sqrt{11}}} dx =$

$\boxed{\frac{3}{5\sqrt{5}}\left[\frac{1}{2}\left(\sqrt{5}x\sqrt{5x^2-11} + 11\ln\left|\sqrt{5}x + \sqrt{5x^2-11}\right|\right)\right] + C}$

2. Solve the differential equation $\frac{dy}{dx} = \frac{y}{x}$, for x and y both greater than zero. And y(1)=5.

$\int \frac{dy}{y} = \int \frac{1}{x}dx$

$\ln y = \ln x + C$

$= \ln x + \ln e^c$

$= \ln(e^c \cdot x)$ let $e^c = A$

$= \ln(Ax)$

$\ln y = \ln(Ax)$

$\boxed{y = Ax}$

$5 = A(1) \Rightarrow A = 5$

$y = 5x$